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Modulation of Schamel's nonlinear Schrödinger equation and related sheath excitations in degenerate relativistic quantum complex plasmas: A review

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ABSTRACT

Degenerate relativistic quantum plasmas are intricate systems in which high-density electrons demonstrate relativistic and quantum mechanical properties. These conditions result in non-linear phenomena, such as soliton production, which are essential for energy transfer and stability analysis. The Schamel nonlinear Schrödinger equation offers a framework for examining wave dynamics in these conditions, integrating fractional non-linearities that elucidate modulation instability and envelope solitons. This study examines the impact of critical physical parameters, including the relativistic degeneracy ratio, ion trapping, and density ratio, on soliton dynamics. The results indicate that the relativistic degeneracy ratio strongly influences wave frequency, group velocity, and energy transport efficiency, particularly in ultra-relativistic plasmas. The research emphasises the mitigating effects of heightened electron degeneracy pressure, which stabilises waves and diminishes soliton amplitude. The vortex-like arrangement of trapped ions enhances the non-linear dynamics of plasma. These findings enhance our comprehension of wave propagation, energy distribution, and stability in extreme plasma conditions.

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1. INTRODUCTION

High-density quantum plasmas in a degenerate state, as observed in extreme astrophysical or laboratory settings, require specialised theoretical techniques. The Schamel nonlinear Schrödinger equation (SNLSE) is a technique employed to characterise wave dynamics in degenerate plasmas. SNLSE integrates aspects of quantum mechanics and non-linear plasma theory, incorporating fractional rank (3/2) non-linearities that facilitate the emergence of envelope soliton solutions with intricate properties. In this condition, electrons can travel at velocities approaching the speed of light, resulting in relativistic effects on their wave structure and propagation characteristics. A degenerate plasma exhibiting relativistic effects demonstrates extremely nonlinear wave interactions, potentially resulting in the emergence of stable localised structures, such as solitons or solitary waves. This phenomena is crucial for comprehending energy distribution in extreme plasma systems [1].

Modulation instability (MI) in relativistic quantum plasma is a noteworthy phenomenon for investigation, as it can generate local wave amplification and localised wave structure. Under these circumstances, relativistic degeneracy effects prevail, resulting in an augmentation of degeneracy pressure and altering the dynamics of interparticle interactions [2]. Employing dual-scale perturbation theory to simplify the SNLSE facilitates a comprehensive examination of the effects of physical parameters, including relativistic degeneracy ratio and ion trapping on soliton stability and propagation.

2. RESEARCH METHODS

2.1. Quantum Relativistic Degeneracy in Plasmas

Relativistic degeneracy in quantum plasmas denotes the state in which electrons experience significant Fermi degeneracy as a result of elevated degeneracy pressure. In this condition, the electrons travel at velocities approaching the speed of light, resulting in the predominance of relativistic effects. The interaction between waves and particles in plasma becomes significantly nonlinear, resulting in MI [3]. In relativistic quantum plasmas, modulational instability typically arises from non-linear interactions between carrier waves and envelope oscillations. This interaction generates localised envelope waves, termed bright envelope solitons, capable of preserving their shape and energy for extended durations. This process is influenced by several physical characteristics, including the relativistic degeneracy ratio, which determines the number of electrons confined in the degenerate state, along with the impact of ions contained inside the plasma. Under these circumstances, the influences of thermodynamics and quantum mechanics significantly impact the stability and dynamics of the plasma, which may differ markedly from that of a classical plasma [4].

Recent research employs a multi-scale perturbation theory technique to formulate a mathematical model that characterises magneto-ionic interactions in relativistic plasmas. Employing this theory, a modified SNLSE can be derived, featuring fractional nonlinearity terms (of rank 3/2) that exhibit greater complexity than the conventional SNLSE equation. The equation representing the relativistic degeneracy pressure for electrons is [5]:

$$P_e = \frac{\pi {m_e}^4 c^5}{3h^3}\left[G\sqrt{1+G^2}(2G^2-3)+\sinh^{-1}G\right] \tag{1}$$

where, G is the relativistic electron denegation parameter.

2.2. SNLSE in a Degenerate Plasma

The SNLSE in degenerate plasmas integrates concepts from quantum mechanics and nonlinear plasma theory to characterise wave dynamics in intricate plasma systems. Degenerate plasmas, composed of high-density electrons in a Fermi degenerate state, display characteristics that markedly differ from those of ordinary plasmas. Under these circumstances, the interaction between electrons and ions frequently exhibits non-linearity, and relativistic effects on the degenerate electrons influence the wave structure and propagation events. The SNLSE employed in this instance incorporates fractional rank (3/2) non-linearity, presenting additional complexities in the examination of wave stability and dynamics, particularly in elucidating the generation and propagation of solitons induced by wave amplitude modulation.

The fractional rank non-linearity in this equation enables the alteration of more intricate and varied wave structures in comparison to simpler plasma models, as anticipated by the classic non-linear Schrödinger equation. In degenerate plasmas, the relativistic attributes of electrons significantly influence wave characteristics, resulting in distinct isolated solutions compared to those observed in plasmas with non-relativistic electrons. Furthermore, phenomena such as trapped ion confinement influence the dynamics of intricate wave-plasma interactions. The waves produced by the system demonstrate stability over a specific duration, enabling them to endure for an extended period to induce sensory effects [6].

The SNLSE enables researchers to examine the dynamics of brilliant envelope waves in complex plasmas, comprising relativistically degenerate electrons, ions, and non-degenerate dust particles. The luminous envelope waves exhibit stability over time, facilitating the examination of energy propagation patterns in degenerate plasmas. This pattern can illustrate the essential traits of the interaction between the electromagnetic field and particles within the plasma. The fundamental expression of SNLSE in this research is:

$$i\frac{\partial\Psi^{(1)}}{\partial T} - \frac{P}{C}\frac{\partial^2\Psi^{(1)}}{\partial T^2} - \frac{Q}{C}\left|\Psi^{(1)}\right|^{\frac{1}{2}}\Psi^{(1)} = 0 \tag{2}$$

where, P and Q are coefficients that denote the contributions of dispersion and non-linearity [4, 7].

2.3. Vortex-Style Distribution of Confined Ions

The vortex-like arrangement of ions confined in a complex plasma is a noteworthy phenomena due to its association with the interaction between electrostatic waves and charged particles within the plasma medium. Complex plasmas comprise charged particles, including electrons, ions, and bigger dust grains, capable of generating electrostatic wave patterns. When ions possessing low kinetic energy are captured by these waves, they will create an atypical distribution structure resembling a vortex pattern. This contrasts with the conventional Maxwellian distribution, which posits that particles are randomly distributed with velocities adhering to a normal distribution. These ions exhibit a vortex-like distribution, congregating in the middle part of the wave, resulting in a higher concentration and circular motion around the centre. The phenomenon can be comprehended more thoroughly by examining the behaviour of particles in intricate plasmas influenced by electrostatic waves. When these waves possess sufficient amplitude, they can modify the kinetic energy of lower ions, resulting in their entrapment in a rotational pattern.

This mechanism generates a vortex structure, wherein ions revolve around a central point, resulting in a distribution pattern that is more centralised than the random dispersion characterised by the Maxwellian distribution. This interaction significantly impacts the examination of plasma wave stability, particularly in dust acoustic waves, which emerge from the interactions among charged ions, electrons, and dust particles. The impact of this vortex-like distribution on the non-linearity and dynamics of plasma waves elucidates numerous phenomena observed in complex plasmas. These non-linearities pertain to alterations in plasma waves that cannot be elucidated by conventional linear equations, frequently resulting from interactions between waves and confined particles. Under these conditions, the distribution of ions confined in the vortex pattern can enhance or diminish the wave amplitude, influencing the development and stability of dust acoustic waves [8].

3. RESULTS AND DISCUSSIONS

3.1. Relativistic MI of the SNLSE

MI in the SNLSE is a significant phenomenon that offers profound understanding of wave propagation and amplification within complex plasmas. SNLSE is employed to represent wave phenomena in plasmas comprising diverse particle types, including relativistically degenerate electrons, confined ions, and undegenerate dust particles. The existence of degenerate electrons in relativistic states substantially influences the system's dynamics, as the relativistic characteristics augment the effective mass of the particles and alter the interactions among them. Consequently, MI in SNLSE enables the examination of how these instabilities can arise inside the distinctive complex plasma medium, where fields and waves interact via nonlinear mechanisms.

The application of the double-scale perturbation method to reduce this equation to an SNLSE with a nonlinear term of fractional order (3/2) produces solutions that demonstrate the characteristic behaviour of modulation instabilities. This instability results in local wave amplification, a mechanism essential for the creation of brilliant envelope soliton structures. These formations are localised manifestations of significantly amplified waves inside the plasma medium, capable of preserving their form during propagation. Under complex plasma conditions, bright envelope solitons are significant as they represent the concentrated energy distribution within the medium, which is crucial for comprehending wave dynamics in plasma environments, such as those surrounding neutron stars or giant planets, where particle interactions and magnetic fields exert considerable influence.

The significance of solitons in energy transport within plasma is a subject of considerable study, particularly with the application of MI to astrophysical plasma settings. These solitons function as an effective means of energy transport within the plasma, yielding considerable implications for the modelling of energy systems in both extraterrestrial and laboratory environments. In the vicinity of neutron stars, intense magnetic fields can alter the behaviour of solitons, resulting in the development of more intricate wave dynamics.

3.2. Impact of Physical Parameters on Acoustic Dust Waves in Complex Plasmas

The impact of physical parameters, particularly the δ parameter, on the dynamics of dust-acoustic waves in complex plasmas is crucial for comprehension, as it affects all facets of plasma technology modelling and applications. The relationship between wave frequency (ω) and group velocity (V_g) with respect to δ indicates that in ultra-relativistic plasmas, little variations in δ can result in significant alterations in the dynamics of acoustic dust waves. Conversely, in weakly relativistic plasmas, fluctuations in δ exert no substantial influence on the waves. An increase in the wave number (k) in ultra-relativistic plasmas will elevate the wave frequency (ω), reflecting the heightened sensitivity of the waves to variations in the plasma environment. This indicates that wave structures in ultra-relativistic plasmas are more influenced by fluctuations in specific physical parameters than those in relativistic plasmas, where wave interactions with dust are less susceptible to variations in these parameters. The parameter δ indicates its effect on the plasma state in both the weak and extreme limits illustrated in Figure 1.

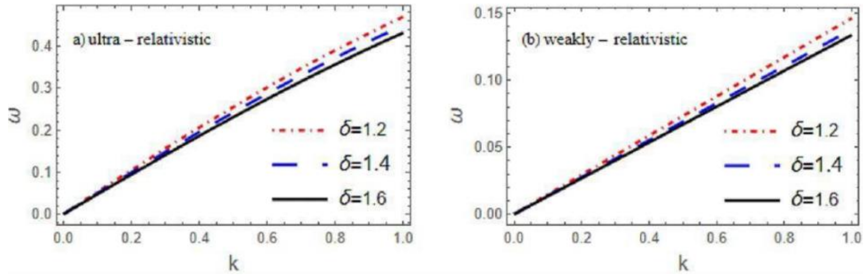

Figure 1. depicts the variation of ω vs k for δ in (a) ultra-relativistic and (b) weakly relativistic plasmas.

In ultra-relativistic plasmas, variations in physical parameters such as δ substantially influence the dynamics of dust-acoustic waves, particularly regarding the group velocity (V_g). In this scenario, a rise in the value of δ results in a more pronounced fluctuation in the group velocity compared to that in the weakly relativistic plasma. This phenomenon can be elucidated by the ultra-relativistic characteristics of the plasma, which render the waves more sensitive to variations in particle energy and momentum. Consequently, an initial increase in the wave number k leads to an enhancement of the group velocity; but, as δ progresses, the group velocity diminishes, signifying a suppression or reversal effect on the wave properties. This suggests that ultra-relativistic plasmas exhibit heightened sensitivity to fluctuations in physical parameters, whereby minor alterations in the value of δ can significantly modify the wave propagation pattern.

Moreover, the interplay between frequency (ω) and group velocity (V_g) with the parameter δ in ultra-relativistic plasmas exhibits intricate dynamics and reciprocal impact. The wave frequency experiences a shift that is more responsive to changes in δ, which influences both the group velocity and indicates the instability in dust-acoustic wave propagation. Under these conditions, an increase in δ often results in a more pronounced damping effect, attributable to heightened contact between plasma particles and the instability induced by fluctuations in physical parameters. These instabilities induce substantial alterations in the energy distribution within the plasma, elevating the likelihood of dampening dust-acoustic waves, particularly under ultra-relativistic conditions that demonstrate heightened sensitivity to variations in the physical environment.

Figure 1 explicitly depicts the influence of the parameter δ on the frequency and group velocity (V_g) in ultra-relativistic plasmas, offering a visual representation of the response of dust-acoustic waves to fluctuations in this parameter. The graph indicates that variations in δ have a more pronounced impact on frequency in ultra-relativistic plasmas compared to weakly relativistic plasmas, where frequency remains relatively steady. The graph indicates that in ultra-relativistic conditions, an increase in δ not only diminishes the frequency value but also influences the group velocity, suggesting that wave propagation is further attenuated as δ rises. Consequently, it can be inferred that the physical parameters in ultra-relativistic plasma exert a more significant influence on the propagation characteristics of dust-acoustic waves, such that minor alterations in these parameters can result in substantial effects on the stability and dynamics of waves within the plasma.

3.3. Influence of Parameter δ on Velocity Group

The influence of the parameter δ on group velocity in plasma exhibits a complex pattern, since Vg, representing the propagation speed of the wave envelope, is notably sensitive to variations in this value. Generally, the group velocity tends to diminish as the value of δ grows, indicating that δ's impact on wave propagation characteristics in the plasma medium will become substantial. When the wave number k is comparatively low, the group velocity attains its maximum, suggesting that under these circumstances, the impact of δ is not very significant. Nevertheless, if the value of k escalates, a more significant reduction in group velocity is observed, particularly in ultra-relativistic plasmas. In this plasma type, minor variations in the value of have a more significant effect on the reduction of Vg compared to weakly relativistic plasmas. This indicates that the relativistic characteristics of ultra-relativistic plasmas enhance the sensitivity of group velocity to changes in δ, hence influencing the overall dynamics of wave propagation in the medium. Figure 2 illustrates the correlation between wavenumber (k) and group velocity (V_g).

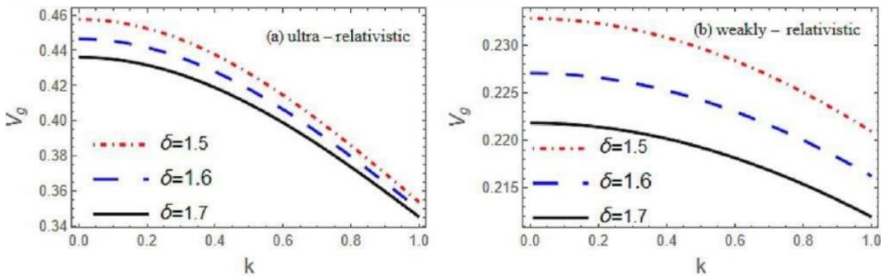

Figure 2. depicts the variation of Vg against k for δ in (a) ultra-relativistic and (b) weakly relativistic plasmas.

In extremely relativistic plasmas, the parameter δ significantly affects the group velocity (V_g) of waves propagating across the medium. This parameter indicates the physical attributes of the system, wherein an elevation in the value of δ signifies a significant relativistic impact on wave behaviour. As the value of δ grows, the response of waves in a relativistic plasma to physical changes in the medium accelerates, leading to a gradual reduction in group velocity. This indicates that under relativistic conditions, an increase in δ might lead to significant alterations in the propagation characteristics of the waves, diminishing their efficiency in sustaining a constant group velocity. The influence of δ under highly relativistic plasma conditions results in a substantial enhancement of V_g damping, hence diminishing the wave's capacity to preserve energy during propagation.

Conversely, in weakly relativistic or less relativistic plasma circumstances, the influence of the δ parameter on Vg is not as pronounced as in relativistic plasma. Under these circumstances, the waves can still sustain a somewhat steady group velocity despite an increase in the value of δ, as the damping effect is not as pronounced as in relativistic plasmas. This indicates that relativistic features significantly influence the group velocity in various plasma types. In plasmas with low relativistic values, wave energy can endure longer during propagation, leading to more stable group velocities. Consequently, comprehending the significance of relativistic effects is crucial for forecasting and regulating wave behaviour in diverse plasma systems with differing physical attributes.

The reduction in group velocity as δ grows can be ascribed to the heightened damping effect in relativistic plasma systems. In ultra-relativistic plasmas, wave energy dissipates more rapidly, leading to a reduction in wave propagation efficiency. Figure 2 illustrates a sharply declining curve for V_g when the δ value is elevated. The phenomena indicates that plasma with a higher δ value can undergo increased damping, hindering the wave's ability to sustain a high group velocity. A larger value of δ corresponds to a more significant damping effect on the wave, which directly diminishes the group velocity and energy propagation efficiency in the relativistic plasma system.

3.4. Impact of Additional Physical Parameters on Soliton Wave Envelopes

In plasma physics, envelope solitons are a noteworthy phenomena because of their ability to preserve shape and energy during propagation, particularly in nonlinear mediums like plasma. These waves are significantly affected by numerous physical elements that govern their stability and properties. An essential metric is the trapping factor (f), which quantifies the strength of ions confined within the plasma field. As the value of f escalates, the quantity of ions confined within the plasma field correspondingly rises. These ions can significantly enhance the nonlinearity of the system, resulting in an augmentation of the amplitude of the envelope soliton wave. The greater the value of f, the more substantial the wave amplitude. This information is crucial for plasma energy regulation, particularly in applications like energy management in fusion systems or plasma-based energy generating devices.

Furthermore, the density ratio factor (δ) influences the envelope soliton wave. An increase in this density ratio, indicating a higher number of plasma particles per unit volume, will result in greater interactions among the particles. The intensity of this interaction enhances the nonlinear energy within the system, hence augmenting the amplitude of the envelope soliton wave. The rise in amplitude signifies that the energy contained in the plasma is elevated and can be sustained in a stable soliton configuration. In this instance, regulating the density ratio offers a proficient method to alter the features of plasma waves, which is crucial for soliton stability in applications like plasma wave communication or nonlinear wave-based remote sensing.

The electron relativistic degeneracy parameter (G_0) has a contrary influence on the envelope soliton wave, enhancing its stability while simultaneously reducing the wave amplitude. As G_0 increases, the influence of electron degeneracy will become more pronounced, hence mitigating the impact of nonlinearity within the system. The rise in pressure causes the plasma to suppress amplitude fluctuations, leading to an envelope soliton wave with diminished amplitude. This suggests that the plasma can more effectively regulate its energy distribution by facilitating rapid energy release in situations characterised by elevated G_0 levels. This action enhances the plasma's stability against external perturbations.

4. CONCLUSION

Relativistic degeneracy in quantum plasmas causes swift electron movement approaching the speed of light, resulting in intricate non-linear phenomena like soliton creation in dense plasma settings. The modification of instability in the SNLSE facilitates the formation of brilliant envelope solitons, which are essential for energy transport in complex plasmas. In ultra-relativistic plasmas, the δ parameter is crucial in ascertaining the frequency and group velocity of acoustic dust waves, where slight alterations can lead to considerable differences. An elevation in may lead to considerable damping, diminishing energy transmission efficacy. Furthermore, parameters such as the density ratio and ion trapping significantly affect the amplitude of envelope soliton waves, rendering them essential for preserving stability and energy properties in the plasma. Increased relativistic degeneracy pressure of electrons mitigates plasma nonlinearity, resulting in reduced soliton amplitude while simultaneously improving wave stability against external perturbations. The vortex-like arrangement of confined ions further illustrates how ion-electrostatic wave interactions can create more organised and centralised patterns, enhancing the plasma's non-linear dynamics.

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