Annual premium of life in insurance with uniform assumptions

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ABSTRACT

This Study discusses life insurance for life with a uniform assumption method. The purpose of this study is to calculate the annual premium that must be paid by life insurance participants. This research was conducted at the General Insurance Group company, which is located on the street Arifin Ahmad, Pekanbaru City, Indonesia. To determine the annual life insurance premium, the interest rate and discount factor, initial lifetime annuity, single premium, and the annual premium. After doing the research, the results of the calculation of the annual premium for life insurance for life using uniform assumptions for each participant life insurance are different. The amount of annual life insurance premiums uses a uniform assumption that is smaller than the annual life insurance premiums obtained from insurance companies.

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1. INTRODUCTION

One effort that can be an alternative to minimize risk is insurance. Whether it's the risk of accidents, death, or illness [1]. Insurance can be the best financial protection and is very useful for its users. The type of insurance that is widely used is life insurance [2, 3]. Life insurance is a service provided by an insurance company in managing risks related to the life or death of an insured person [4]. There are two life insurance developments in Indonesia, namely individual life insurance and group life insurance [5, 6]. The difference between the two insurance lies in the number of dependents. For individual life insurance, the number of insured is only one person or single, while in group life insurance the insurance company bears two or more dependents [7].

Based on the term and protection of life insurance is divided into three, namely life insurance, life insurance dual purpose and term life insurance [8]. The insurance used in this thesis is life insurance, which is life insurance that protects the insured for the rest of his life or until the insured dies [9]. Insurance companies usually agree the insured and the guarantor regarding their respective rights and obligations [10]. The insurance company will charge a premium to be paid by the insured. Which premiums must be paid have been estimated in advance or have been calculated based on the value of the risk to be faced. The greater the risk, the greater the premium to be paid by the insured. There are several types of life insurance premiums, including single premium and annual premium.

Based on the premium used in this thesis is the annual premium which is to determine the amount of the annual premium to be paid requires a single premium and the initial life annuity value, which is influenced by the chance of life and the chance of dying. For life insurance in determining the amount of the annual premium used is life-saving annuity [11].

The annual premium using uniform assumptions has been previously researched by Pinawati (2017) with the title annual premiums for dual life insurance with uniform assumptions [12]. In this

study, the insurance used is dual life insurance. Research related to life insurance has also been carried out by Sesilia (2016) with the title annual premium life insurance for life with de Moivre law [13]. In this study, the method used is de Moivre law using life insurance for life. Based on this, the authors are interested in conducting research and pouring it into the final project with the title annual premium life insurance for life with uniform assumptions.

2. RESEARCH METHODS

In this article, the author discusses the calculation of the annual premium for life insurance. Life insurance is that protects the insured for the rest of his life or until the insured dies.

2.1. Life Opportunities and Death Opportunities

According to Landriault and Willmot (2009), the function of survival is the basis in determining the chances of life and the chances of someone's death [14]. Let x be the age of someone who is still alive, with $x \geq 0$. The life opportunity of someone who is x to t years can be denoted by ${}_t p_x$. So for the equality of life opportunities of people aged x to t years can be stated with:

$$
{}_t p_x = 1 - {}_t q_x \tag{1}
$$

whereas for the chance of death, a person aged x *to* t years can be denoted by ${}_t q_x$. So for the equation of the probability of dying someone aged x to t years is as follows:

$$
{}_t q_x = 1 - {}_t p_x \tag{2}
$$

2.2. Uniform Assumption Methods

Uniform assumptions are assumptions that state that the exit opportunities for insurance participants at all times are the same. According to Landriault and Willmot (2009) the chance of leaving someone aged x years to t the following year at intervals $0 \leq t < 1$ is denoted by ${}_t q_x$ using a uniform assumption expressed as follows [14]:

$$
{}_t q_x = {}_t q_x \tag{3}
$$

then by substituting Equation (3) into (1), life opportunity obtained from someone aged x years to t the following year assuming the life opportunity for insurance participants at each time is the same:

$$
{}_t p_x = 1 - {}_t q_x \tag{4}
$$

this assumption will be used in determining annuity and life insurance premiums.

2.3. Interest Rates and Discount Factors

The interest rate is one of the things that is very influential in determining premiums on life insurance. Interest is part of the company's profits because even in premium payments the element of interest will also be calculated. That is the ratio between interest earned against invested capital, symbolized by i. Interest rates are calculated based on the initial capital that has been added to the interest periodically, then capital added to the interest is returned interest is called the compound interest rate. The user is the compound interest rate. Compound interest rates are defined as a function, namely the discount factor, and notated with [14]:

$$
v = \frac{1}{1+i} \tag{5}
$$

2.4. Life Annuity

The life annuity is divided into two types namely term life annuity and a lifetime annuity. Those that are only valid for a certain period are called term life annuities. While annuities that apply throughout the life of the insured or payment will cease if the insured dies are called a lifetime annuity. Life annuity payments do so at the beginning of each year and some at the end of the policy year. It is

known that $_tp_x$ states the life opportunity of life insurance participants aged x years to t years, ω represents the maximum age of life insurance participants and v states the discount factor, and payment is made at the beginning of the policy year. So the value of an initial lifetime annuity [11]:

$$\ddot{a}_x = 1 + vp_x + v^2{}_2p_x + v^3{}_3p_x + \cdots + v^{\omega-x-1}{}_{\omega-x-1}p_x = \sum_{t=0}^{\omega-x-1} v^t{}_tp_x \tag{6}$$

2.5. Single Premium and Annual Premium

According to Futami (1993), a single premium is an official payment of insurance made at the time the insurance contract is approved and there is no further payment [1]. While annual premiums are premiums that are paid at the beginning of each year, the amount of which can be the same or change. Before determining the annual premium amount we must first determine a single premium for life insurance. A single-life insurance premium with $n = \omega - x$ coverage period is stated as a single premium for life insurance that can be denoted as A_x [15-18]. For example ω states the maximum age of the insurance participant, $_tp_x$ states the life opportunity of the life insurance participant who is x years old lives up to t years and will die 1 year later stated by:

$$A_x = \sum_{t=0}^{\omega-x-1} v^{t+1}{}_{t|}q_x \tag{7}$$

Next, determine the annual premium for life insurance, for example A_x, declare a single premium for life insurance, $\ddot{a}_x$ declare the initial life annuity for life insurance, then the annual premium for life insurance is stated by [19]:

$$P_x = R\frac{A_x}{\ddot{a}_x} \tag{8}$$

3. RESULTS AND DISCUSSION

3.1. Early Life Annuity with Uniform Assumptions

The cash value of an initial annuity for life is the cash value of an annuity that is affected by the discount factor and the probability of death and is calculated at the beginning of the period for years, symbolized by $\ddot{a}_x$, with x which is the age of the insurance participant [20]. By substituting Equation (4) uniform assumptions with Equation (6), the cash value of the initial lifetime annuity is:

$$\ddot{a}_x = \sum_{t=0}^{\omega-x-1} v^t{}_tp_x = \sum_{t=0}^{\omega-x-1} v^t(1 - tq_x) \tag{9}$$

then, a lifetime initial annuity is obtained by uniform assumptions.

3.2. Single Premium Life Insurance for Life with Uniform Assumptions

The single premium for life insurance for insurance participants aged x years with a period of coverage for t years and the sum insured is paid at the end of the policy notated A_x [21]. By substituting Equation (4) uniform assumptions with Equation (7), a single premium for life insurance:

$$A_x = \sum_{t=0}^{\omega-x-1} v^{t+1}{}_{t|}q_x = \sum_{t=0}^{\omega-x-1} v^{t+1}(_tp_x q_{x+t}) = \sum_{t=0}^{\omega-x-1} v^{t+1}((1 - tq_x)(q_{x+t})) \tag{10}$$

then, obtained a single premium for life insurance with uniform assumptions.

3.3. Annual Life Insurance Premiums for Life with Uniform Assumptions

Life insurance annual premium for insurance participants aged x years with a period of coverage for t years and premium payments made until the end [11]. By substituting Equation (4) uniform assumptions with Equation (8) the annual premium for life insurance can be stated by:

$$P_x = R\frac{A_x}{\ddot{a}_x} = R\frac{\sum_{t=0}^{\omega-x-1} v^{t+1}((1-tq_x)(q_{x+t}))}{\sum_{t=0}^{\omega-x-1} v^t(1-tq_x)} \tag{11}$$

then, get an annual premium for life insurance for life with Uniform Assumptions.

3.4. Case Study

The data to be processed is the life insurance participant data where participants have different ages and different amounts of coverage. The maximum age for male insurance participants is 100 years, while the maximum age for female insurance participants is 103 years, with a 10% interest rate for each period. From these data, the author will determine the amount of the annual premium for life insurance for life by using a uniform assumption. Before determining the annual life insurance premium amount, we must first determine the discount factor with an interest rate of 10%.

$$v = \frac{1}{1+i} = \frac{1}{1+0,1} = 0,9090$$

Next, a lifetime initial annuity cash value will be determined by the Uniform Assumption, a single premium with the Uniform Assumption, and the amount of the annual life insurance premium for the Uniform Assumption for each data.

3.5. Example

Mr. Zulkifli is someone who is 59 years old and wants to take life insurance for life with an interest of 10% and a sum of 150,000,000 IDR then the annual premium to be paid by Mr. Zulkifli using the Uniform Assumption where the estimated maximum age is 100 years, the results are obtained. From the example above obtained and can be known:

$$x = 59 \text{ years old}, \omega = 100 \text{ years old}, R = 150,000,000 \text{ IDR}$$

Opportunities for life insurance participants who are 59 years old for the next 0 years are:

$$_tq_{59} = {_0}q_{59} = 0$$

The chance of dying a life insurance participant aged 59 years for the next 1 year is determined, then obtained:

$$_tq_x = \frac{l_x - l_{x+t}}{l_x}$$

$$_tq_{59} = {_1}q_{59} = \frac{l_{59} - l_{59+1}}{l_{59}} = 1 - \frac{l_{60}}{l_{59}} = \left(1 - \frac{83734}{84896}\right) = 0.0136873$$

The probability of dying a life insurance participant aged 59 years for the next 2 years is determined, then obtained:

$$_tq_x = tq_x$$

$$_2q_{59} = 2q_{59} = 0.0264819$$

$$\vdots$$

$$_{40}q_{59} = 40q_{59} = 0.9978326$$

Next, determine the initial lifetime lifetime annuity cash value from Equation (9), then we get:

$$\ddot{a}_{59} = \sum_{t=0}^{\omega-x-1} v^t (1 - tq_x)$$

$$\ddot{a}_{59} = ((1 + 0.9090(1 - 0.0136873) + 0,9090^2(1 - 0.0284819) + \cdots + 0.9090^{40}(1 - 0.9978326)$$

$$\ddot{a}_{59} = 8.4408048$$

After determining the initial annuity value for life then determine the single premium from Equation (10), we obtain:

$$A_{59} = \sum_{t=0}^{\omega-x-1} v^{t+1}((1-tq_x)(q_{x+t}))$$

$$A_{59} = 0.9090(0.0136873) + 0.9090^2(1 - 0.0136878)(0.0149991) + \cdots + 0.90^{9041}(1 - 0.9978326)(0.4673930)$$

$$A_{59} = 0.231897$$

so the value of a single premium for life insurance with 59 years of age is 0.231897. If multiplied by the sum of 150,000,000 IDR the amount of the single premium to be paid is 34,784,550 IDR. Furthermore, using Equation (11) an annual life insurance premium value is obtained as follows:

$$P_x = R \frac{\sum_{t=0}^{\omega-x-1} v^{t+1}((1-tq_x)(q_{x+t}))}{\sum_{t=0}^{\omega-x-1} v^t (1-tq_x)}$$

$$P_{59} = 150,000,000 \times \frac{0,231897}{8,4408048} = 4,120,999$$

so, the annual premium to be paid by Zulkifli with an age of 59 years of insurance is 4,120,999 IDR.

The following is a comparison of the annual premium for life insurance using the Assumption Uniform with the annual premium for life insurance determined by the company as shown in Table 1. Based on the Table 1, it can be seen that the amount of the annual premium for life insurance for life using a uniform assumption is smaller when compared to the annual premium obtained from the company [16, 22]. This is due to insurance companies in determining the number of premiums to be paid by the insured the company considers the level of salary, medical history, and the work of the insured [23-25]. While uniform assumptions do not look at it.

Table 1. Comparison of annual life insurance premiums for life.

Age	Gender	Sum insured (IDR)	Company annual premium (IDR)	Uniform annual premium assumptions (IDR)
59 th	Male	150,000,000	4,500,000	4,120,999
54 th	Male	70,000,000	3,500,000	1,330,164
58 th	Male	150,000,000	4,500,000	3,147,160
51 th	Male	100,000,000	4,000,000	1,558,069
52 th	Male	70,000,000	2,000,000	1,189,990
53 th	Male	150,000,000	3,000,000	2,634,573
55 th	Female	200,000,000	5,000,000	3,011,177
51 th	Male	70,000,000	1,500,000	1,090,648
53 th	Male	100,000,000	3,000,000	1,756,382
59 th	Male	75,000,000	2,500,000	2,060,499
55 th	Female	70,000,000	2,000,000	1,053,912
54 th	Male	100,000,000	3,000,000	1,900,234
52 th	Male	200,000,000	5,000,000	3,399,973
58 th	Male	150,000,000	4,200,000	2,098,107
52 th	Male	100,000,000	3,600,000	1,699,986

4. CONCLUSION

The amount of the annual life insurance premium is determined by the age of the insurance participant, the interest rate, the discount factor, the initial life annuity for life, and the amount of the sum insured. The data applied to calculate the annual life insurance premium is 15 insurance participants, with a different sum insured and an interest of 10%. Based on the results obtained by the author that the results of the calculation of the amount of the annual premium to be paid by each participant for life insurance using a uniform assumption varies. The amount of the annual premium for life insurance for life by using a uniform assumption is smaller than the annual premium determined by the company. Thus this method can be used as a reference to calculate the amount of the annual life insurance premium.

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