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On (*f*, *g*)-derivation in BCH-algebra

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ABSTRACT

BCH-algebra is a non-empty set with the binary operation $*$ and the constant 0, and statisfying the certain axioms. A mapping of d from X to itself is said to be a derivation in BCH-algebra if d is both (l, r)-derivation and (r, l)-derivation in BCH-algebra, where X is BCH-algebra. This article discusses the concepts of (l, r)-(f, g)-derivation, (r, l)-(f, g)-derivation, and (f, g)-derivation in BCH-algebra, and investigates the properties (l, r)-(f, g)-derivation, (r, l)-(f, g)-derivation and (f, g)-derivation in BCH-algebra.

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1. INTRODUCTION

Hu and Li [1] introduced an algebraic structure called BCH-algebra, which is a generalization of BCK-algebra and BCI-algebra. BCH-algebra is a non-empty set X with the binary operation * and the constant 0, which satisfies the axiom (BCH1) $x * x = 0$, (BCH2) $x \leq y$ and $y \leq x$ imply $x = y$, where $x \leq y$ if and only if $x * y = 0$, and (BCH3) $(x * y) * z = (x * z) * y$ for any $x, y \in X$ [2, 3].

The concepts of BCH-algebra have been discussed by researchers, for instance the concept of derivation [4-9]. The derivation in BCI-algebra was introduced by Jun and Xin [10]. In 2005 Zhan and Liu [11] introduced the notion of derivation in BCI-algebra to find the concept of f-derivation in BCI-algebra, where f is the endomorphism in BCI-algebra [12-21]. Al-shehri [22] applied the concept of the derivation of BCI-algebra to B-algebra in 2010, then the notion was introduced by Ardekani and Davvas [23] in order to obtain a new idea, called (f, g)-derivation in B-algebra, where f and g is an endomorphism of B-algebra. In 2015, Bawazeer and Bashammakh [24] introduced the notion of derivation in BCH-algebra and fixed sets. The notion of derivation defined the concept of derivation in BCH-algebra. A mapping d from X to itself is called to be a derivation in BCH-algebra if d is both an (l, r)-derivation and (r, l)-derivation in BCH-algebra, with X is BCH-algebra.

The objective of this paper is to define (f, g)-derivation in BCH-algebra and investigate some of the properties of (f, g)-derivation in BCH-algebra.

2. PRELIMINARIES

In this section, the definition and some properties of BCH-algebra are given.

2.1. Definition 1

BCH-algebra is an algebra $(X; *, 0)$ that satisfies the following axioms [11], for all $x, y \in X$:

(BCH1) $x * x = 0$
(BCH2) $x \leq y$ and $y \leq x$ imply $x = y$, where $x \leq y$ if and only if $x * y = 0$
(BCH3) $(x * y) * z = (x * z) * y$

In BCH-algebra $(X; *, 0)$ the following proporties apply, for all $x, y \in X$:

(BCH4) $(x * (x * y)) * y = 0$
(BCH5) $x * 0 = 0$ implies $x = 0$
(BCH6) $0 * (x * y) = (0 * x) * (0 * y)$
(BCH7) $x * 0 = x$
(BCH8) $(x * y) * x = 0 * y$
(BCH9) $x * y = 0$ implies $0 * x = 0 * y$
(BCH10) $x * (x * y) \leq y$

Example, let $X = \{0,1,2\}$ be a set with *Cayley* table as follows,

Table 1. *Cayley* table for $(X; *, 0)$.

$*$	0	1	2	3
0	0	0	2	2
1	1	0	2	2
2	2	2	0	0
3	3	2	1	0

From Table 1 it can be shown that $(X; *, 0)$ satisfying all axioms BCH-algebra, so that $(X; *, 0)$ is BCI-algebra.

For a BCH-algebra $(X; *, 0)$, we denote $x \wedge y = y * (y * x)$. A mapping f of a BCH-algebra X into itself is called an endomorphism of X if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$. Note that $f(0) = 0$.

2.2. Definition 2

Let $(X; *, 0)$be a BCH-algebra and d is a mapping from X into itself [16]. The d mapping is to be (l, r)-derivation in X, if it satisfies $d(x * y) = (d(x) * y)) \wedge (x * d(y))$ for all $x, y \in X$ and d called (r, l)-derivation in X if it satisfies $d(x * y) = (x * d(y)) \wedge (d(x) * y)$ for all $x, y \in X$. Moreover, If d is both an (l, r)-derivation and (r, l)-derivation, then d is derivation in X.

2.3. Definition 3

Let $(X; *, 0)$ be a BCH-algebra and d is a mapping from X into itself [24]. The d mapping is called to be regular if $d(0) = 0$.

3. RESULTS AND DISCUSSION

This section consider the main results of research, which is to define (f, g)-derivation in BCH-Algebra, where f and g are endomorphisms of BCH-Algebra [25-28]. Then, define (f, g)-derivation in BCH-algebra and construct its properties which are expressed in proposition form.

3.1. Definition (f, g)-Derivation in BCH-Algebra

Let $(X; *, 0)$ be a BCH-algebra and d is a mapping from X to itself, where f, g is an endomorphism of X. The d mapping is said to be (l, r)-(f, g)-derivation in X if it satisfies $d(x * y) = (d(x) * f(y)) \wedge (g(x) * d(y))$ for all $x, y \in X$ and d called (r, l)-(f, g)-derivation in X if it satisfies $d(x * y) = (f(x) * d(y)) \wedge (d(x) * g(y))$ for all $x, y \in X$. Moreover, if d is both an (l, r)-(f, g)-derivation and (r, l)-(f, g)-derivation, then d is(f, g)-derivation in X.

Example, let $X = \{0,1,2\}$ be a set defined in Table 2, Tabel 2. Tabel *Cayley* untuk $(X;*,0)$.

$*$	0	1	2
0	0	0	2
1	1	0	2
2	2	2	0

It can be shown that $(X;*,0)$, satisfies all BCH-algebra axioms such that $(X;*,0)$. Define a map $d,f,g: X \to X$ by:

$$d(x)=\begin{cases}0 & if\ x=0,1\\ 2 & if\ x=2\end{cases},\qquad f(x)=\begin{cases}0 & if\ x=0,1\\ 2 & if\ x=2\end{cases},\qquad g(x)=\begin{cases}0 & x=0\\ 1 & x=1\\ 2 & x=2\end{cases}$$

So it can be proved that f and g are endomorphisms of X and d is (f,g)-derivation in X.

3.1.1. Proposition 1

Let $(X;*,0)$ be a BCH-algebra, d is a self-map of X, where f,g is an endomorphism of X.

1. Let d be (l,r)-(f,g)-derivation in X. If d regular then $d(x)=d(x)\wedge g(x)$, for all $x\in X$.
2. Let d be (r,l)-(f,g)-derivation in X. d regular if and only if $d(x)=f(x)\wedge d(x)$, for all $x\in X$.

Proof,

1. Let d is (l,r)-(f,g)-derivation in X. Since d regular, then $d(0)=0$, and by axiom (BCH7):

$$d(x)=d(x*0)$$
$$d(x)=(d(x)*f(0))\ \wedge(g(x)*d(0))$$
$$d(x)=(d(x)*0)\ \wedge(g(x)*0)$$
$$d(x)=d(x)\wedge g(x)$$

Hence, this shows that $d(x)=d(x)\wedge g(x)$, for all $x\in X$.

2. Let d is(r,l)-(f,g)-derivation in X. Since d regular, then $d(0)=0$, and by axiom (BCH7):

$$d(x)=d(x*0)$$
$$d(x)=(f(x)*d(0))\ \wedge(d(x)*g(0))$$
$$d(x)=(f(x)*0)\ \wedge(d(x)*0)$$
$$d(x)=f(x)\wedge d(x)$$

Hence, this shows that $d(x)=f(x)\wedge d(x)$for all $x\in X$. Conversely, let d is (r,l)-(f,g)-derivation in X. If $d(x)=f(x)\wedge d(x)$, then for $x=0$, we have:

$$d(0)=(f(0)\wedge d(0))$$
$$d(0)=(0\wedge d(0))$$
$$d(0)=d(0)*(d(0)*0)$$
$$d(0)=d(0)*d(0)$$
$$d(0)=0$$

Hence, this shows that d is regular.

3.1.2. Proposition 2

Let $(X;*,0)$ be a BCH-algebra.

1. If d is a (l,r)-(f,g)-derivation in X, then $d(x*y)\le d(x)*f(y)$, for all $x,y\in X$.
2. If d is a (r,l)-(f,g)-derivation in X, then $d(x*y)\le f(x)*d(y)$, for all $x,y\in X$.

Proof,

1. Since d is a (l,r)-(f,g)-derivation in X and by axiom (BCH10) we have:

$d(x*y) = (d(x)*f(y)) \wedge (g(x)*d(y))$
$d(x*y) = (g(x)*d(y))*(g(x)*d(y))*(d(x)*f(y))$
$d(x*y) \leq d(x)*f(y)$

Hence, this shows that $d(x*y) \leq d(x)*f(y)$ for all $x, y \in X$.

2. Since d is a (r,l)-(f,g)-derivation in X and by axiom (BCH10) we have:

$d(x*y) = (f(x)*d(y) \wedge (d(x)*g(y))$
$d(x*y) = \big(d(x)*g(y)\big)*(\big(d(x)*g(y)\big)*(f(x)*d(y))$
$d(x*y) \leq f(x)*d(y)$

Hence, this shows that $d(x*y) \leq f(x)*d(y)$ for all $x, y \in X$.

3.1.3. Proposition 3

Let $(X;*,0)$ be a BCH-algebra.

1. If d is a (r,l)-(f,g)-derivation in X and d regular, then $d(x) \leq f(x)$ for all $x \in X$.

Proof,

1. Let d is a (r,l)-(f,g)-derivation in X. Since d regular, then $d(0) = 0$. By axiom (BCH7) and (BCH10), for all $x \in X$ we obtained:

$d(x) = d(x*0)$
$d(x) = (f(x)*d(0)) \wedge (d(x)*g(0))$
$d(x) = f(x) \wedge d(x)$
$d(x) = d(x)*(d(x)*f(x))$
$d(x) \leq f(x)$

Hence, $d_f(x) \leq f(x)$ for all $x \in X$.

3.1.4. Proposition 4

Let $(X;*,0)$ be a BCH-algebra, d is a mapping from X to itself.

1. Let d be (r,l)-(f,g)-derivation in X. If $f(x)*d(x) = 0$ for all $x \in X$, then d regular.
2. Let d be (l,r)-(f,g)-derivation in X. If $d(x)*f(x) = 0$ for all $x \in X$, then d regular.

Proof,

1. Let d is (r,l)-(f,g)-derivation in X. By axiom (BCH1) and (BCH7), for all $x \in X$ obtained:

$d(0) = d(x*x)$
$d(0) = (f(x)*d(x)) \wedge (d(x)*g(x))$
$d(0) = 0 \wedge (d(x)*g(x))$
$d(0) = \big(d(x)*g(x)\big)*((d(x)*g(x))*0)$
$d(0) = \big(d(x)*g(x)\big)*(d(x)*g(x))$
$d(0) = 0$

Hence, $d(0) = 0$ then it is proved that d regular.

2. Let d is (l,r)-(f,g)-derivation in X. By axiom (BCH1) and (BCH7), for all $x \in X$ obtained:

$d(0) = d(x*x)$
$d(0) = (d(x)*f(x)) \wedge (g(x)*d(x))$
$d(0) = 0 \wedge (g(x)*d(x))$
$d(0) = \big(g(x)*d(x)\big)*((g(x)*d(x))*0)$
$d(0) = \big(g(x)*d(x)\big)*(g(x)*d(x))$
$d(0) = 0$

Hence, $d(0) = 0$ then it is proved that d regular.

4. CONCLUSION

This article discussed the definition of (l, r) and (r, l)-(f, g)-derivation in BCH-algebra and investigate its proporties. In general, the properties of (l, r) and (r, l)-(f, g)-derivation in BCH-algebra obtained for d which satisfies the regular properties that is when $d(0) = 0$.

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