SINTECHCOM: Science, Technology, and Communication Journal
Vol. 1, No. 3, June 2021, pp. 89-94, DOI: 10.59190/stc.v1i3.196

On (*f*, *g*)-derivation in BCH-algebra

Sherly Afriastuti, Sri Gemawati*, Syamsudhuha
Department of Mathematics, Universitas Riau, Pekanbaru 28293, Indonesia

ABSTRACT

BCH-algebra is a non-empty set with the binary operation $*$ and the constant 0, and statisfying the certain axioms. A mapping of *d* from *X* to itself is said to be a derivation in BCH-algebra if *d* is both (*l*, *r*)-derivation and (*r*, *l*)-derivation in BCH-algebra, where *X* is BCH-algebra. This article discusses the concepts of (*l*, *r*)-(*f*, *g*)-derivation, (*r*, *l*)-(*f*, *g*)-derivation, and (*f*, *g*)-derivation in BCH-algebra, and investigates the properties (*l*, *r*)-(*f*, *g*)-derivation, (*r*, *l*)-(*f*, *g*)-derivation and (*f*, *g*)-derivation in BCH-algebra.

ARTICLE INFO

Article history:

Received Apr 16, 2021
Revised Apr 30, 2021
Accepted May 14, 2021

Keywords:

Axiom
BCH-Algebra
(*f*, *g*)-Derivation
(*l*, *r*)-(*f*, *g*)-Derivation
(*r*, *l*)-(*f*, *g*)-Derivation

* **Corresponding Author**
E-mail address: germawati.sri@gmail.com

1. INTRODUCTION

Hu and Li [1] introduced an algebraic structure called BCH-algebra, which is a generalization of BCK-algebra and BCI-algebra. BCH-algebra is a non-empty set X with the binary operation * and the constant 0, which satisfies the axiom (BCH1) $x * x = 0$, (BCH2) $x \leq y$ and $y \leq x$ imply $x = y$, where $x \leq y$ if and only if $x * y = 0$, and (BCH3) $(x * y) * z = (x * z) * y$ for any $x, y \in X$ [2, 3].

The concepts of BCH-algebra have been discussed by researchers, for instance the concept of derivation [4-9]. The derivation in BCI-algebra was introduced by Jun and Xin [10]. In 2005 Zhan and Liu [11] introduced the notion of derivation in BCI-algebra to find the concept of f-derivation in BCI-algebra, where f is the endomorphism in BCI-algebra [12-21]. Al-shehri [22] applied the concept of the derivation of BCI-algebra to B-algebra in 2010, then the notion was introduced by Ardekani and Davvas [23] in order to obtain a new idea, called (f, g)-derivation in B-algebra, where f and g is an endomorphism of B-algebra. In 2015, Bawazeer and Bashammakh [24] introduced the notion of derivation in BCH-algebra and fixed sets. The notion of derivation defined the concept of derivation in BCH-algebra. A mapping d from X to itself is called to be a derivation in BCH-algebra if d is both an (l, r)-derivation and (r, l)-derivation in BCH-algebra, with X is BCH-algebra.

The objective of this paper is to define (f, g)-derivation in BCH-algebra and investigate some of the properties of (f, g)-derivation in BCH-algebra.

2. PRELIMINARIES

In this section, the definition and some properties of BCH-algebra are given.

2.1. Definition 1

BCH-algebra is an algebra $(X; *, 0)$ that satisfies the following axioms [11], for all $x, y \in X$:

(BCH1) $x * x = 0$
(BCH2) $x \leq y$ and $y \leq x$ imply $x = y$, where $x \leq y$ if and only if $x * y = 0$
(BCH3) $(x * y) * z = (x * z) * y$

In BCH-algebra $(X; * ,0)$ the following proporties apply, for all $x, y \in X$:

(BCH4) $(x * (x * y)) * y = 0$
(BCH5) $x * 0 = 0$ implies $x = 0$
(BCH6) $0 * (x * y) = (0 * x) * (0 * y)$
(BCH7) $x * 0 = x$
(BCH8) $(x * y) * x = 0 * y$
(BCH9) $x * y = 0$ implies $0 * x = 0 * y$
(BCH10) $x * (x * y) \leq y$

Example, let $X = \{0,1,2\}$ be a set with *Cayley* table as follows,

Table 1. *Cayley* table for $(X; * ,0)$.

$*$	0	1	2	3
0	0	0	2	2
1	1	0	2	2
2	2	2	0	0
3	3	2	1	0

From Table 1 it can be shown that $(X; * ,0)$ satisfying all axioms BCH-algebra, so that $(X; * ,0)$ is BCI-algebra.

For a BCH-algebra $(X; * ,0)$, we denote $x \wedge y = y * (y * x)$. A mapping f of a BCH-algebra X into itself is called an endomorphism of X if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$. Note that $f(0) = 0$.

2.2. Definition 2

Let $(X; * ,0)$be a BCH-algebra and d is a mapping from X into itself [16]. The d mapping is to be (l, r)-derivation in X, if it satisfies $d(x * y) = (d(x) * y)) \wedge (x * d(y))$ for all $x, y \in X$ and d called (r, l)-derivation in X if it satisfies $d(x * y) = (x * d(y)) \wedge (d(x) * y)$ for all $x, y \in X$. Moreover, If d is both an (l, r)-derivation and (r, l)-derivation, then d is derivation in X.

2.3. Definition 3

Let $(X; * ,0)$ be a BCH-algebra and d is a mapping from X into itself [24]. The d mapping is called to be regular if $d(0) = 0$.

3. RESULTS AND DISCUSSION

This section consider the main results of research, which is to define (f, g)-derivation in BCH-Algebra, where f and g are endomorphisms of BCH-Algebra [25-28]. Then, define (f, g)-derivation in BCH-algebra and construct its properties which are expressed in proposition form.

3.1. Definition (f, g)-Derivation in BCH-Algebra

Let $(X; * ,0)$ be a BCH-algebra and d is a mapping from X to itself, where f, g is an endomorphism of X. The d mapping is said to be (l, r)-(f, g)-derivation in X if it satisfies $d(x * y) = (d(x) * f(y)) \wedge (g(x) * d(y))$ for all $x, y \in X$ and d called (r, l)-(f, g)-derivation in X if it satisfies $d(x * y) = (f(x) * d(y)) \wedge (d(x) * g(y))$ for all $x, y \in X$. Moreover, if d is both an (l, r)-(f, g)-derivation and (r, l)-(f, g)-derivation, then d is(f, g)-derivation in X.

Example, let $X = \{0,1,2\}$ be a set defined in Table 2, Tabel 2. Tabel *Cayley* untuk $(X; *, 0)$.

$*$	0	1	2
0	0	0	2
1	1	0	2
2	2	2	0

It can be shown that $(X; *, 0)$, satisfies all BCH-algebra axioms such that $(X; *, 0)$. Define a map $d, f, g: X \to X$ by:

$$d(x) = \begin{cases} 0 & if\ x = 0,1 \\ 2 & if\ x = 2 \end{cases}, \qquad f(x) = \begin{cases} 0 & if\ x = 0,1 \\ 2 & if\ x = 2 \end{cases}, \qquad g(x) = \begin{cases} 0 & x = 0 \\ 1 & x = 1 \\ 2 & x = 2 \end{cases}$$

So it can be proved that f and g are endomorphisms of X and d is (f, g)-derivation in X.

3.1.1. Proposition 1

Let $(X; *, 0)$ be a BCH-algebra, d is a self-map of X, where f, g is an endomorphism of X.

1. Let d be (l, r)-(f, g)-derivation in X. If d regular then $d(x) = d(x) \wedge g(x)$, for all $x \in X$.
2. Let d be (r, l)-(f, g)-derivation in X. d regular if and only if $d(x) = f(x) \wedge d(x)$, for all $x \in X$.

Proof,

1. Let d is (l, r)-(f, g)-derivation in X. Since d regular, then $d(0) = 0$, and by axiom (BCH7):

$$\begin{aligned} d(x) &= d(x * 0) \\ d(x) &= (d(x) * f(0)) \wedge (g(x) * d(0)) \\ d(x) &= (d(x) * 0) \wedge (g(x) * 0) \\ d(x) &= d(x) \wedge g(x) \end{aligned}$$

Hence, this shows that $d(x) = d(x) \wedge g(x)$, for all $x \in X$.

2. Let d is(r, l)-(f, g)-derivation in X. Since d regular, then $d(0) = 0$, and by axiom (BCH7):

$$\begin{aligned} d(x) &= d(x * 0) \\ d(x) &= (f(x) * d(0)) \wedge (d(x) * g(0)) \\ d(x) &= (f(x) * 0) \wedge (d(x) * 0) \\ d(x) &= f(x) \wedge d(x) \end{aligned}$$

Hence, this shows that $d(x) = f(x) \wedge d(x)$for all $x \in X$. Conversely, let d is (r, l)-(f, g)-derivation in X. If $d(x) = f(x) \wedge d(x)$, then for $x = 0$, we have:

$$\begin{aligned} d(0) &= (f(0) \wedge d(0)) \\ d(0) &= (0 \wedge d(0)) \\ d(0) &= d(0) * (d(0) * 0) \\ d(0) &= d(0) * d(0) \\ d(0) &= 0 \end{aligned}$$

Hence, this shows that d is regular.

3.1.2. Proposition 2

Let $(X; *, 0)$ be a BCH-algebra.

1. If d is a (l, r)-(f, g)-derivation in X, then $d(x * y) \leq d(x) * f(y)$, for all $x, y \in X$.
2. If d is a (r, l)-(f, g)-derivation in X, then $d(x * y) \leq f(x) * d(y)$, for all $x, y \in X$.

Proof,

1. Since d is a (l, r)-(f, g)-derivation in X and by axiom (BCH10) we have:

$d(x * y) = (d(x) * f(y)) \wedge (g(x) * d(y))$
$d(x * y) = (g(x) * d(y)) * (g(x) * d(y)) * (d(x) * f(y))$
$d(x * y) \leq d(x) * f(y)$

Hence, this shows that $d(x * y) \leq d(x) * f(y)$ for all $x, y \in X$.

2. Since d is a (r, l)-(f, g)-derivation in X and by axiom (BCH10) we have:

$d(x * y) = (f(x) * d(y) \wedge (d(x) * g(y))$
$d(x * y) = \big(d(x) * g(y)\big) * ((\, d(x) * g(y)\big) * (f(x) * d(y))$
$d(x * y) \leq f(x) * d(y)$

Hence, this shows that $d(x * y) \leq f(x) * d(y)$ for all $x, y \in X$.

3.1.3. Proposition 3

Let $(X; *, 0)$ be a BCH-algebra.

1. If d is a (r, l)-(f, g)-derivation in X and d regular, then $d(x) \leq f(x)$ for all $x \in X$.

Proof,

1. Let d is a (r, l)- (f, g)-derivation in X. Since d regular, then $d(0) = 0$. By axiom (BCH7) and (BCH10), for all $x \in X$ we obtained:

$d(x) = d(x * 0)$
$d(x) = (f(x) * d(0)) \wedge (d(x) * g(0))$
$d(x) = f(x) \wedge d(x)$
$d(x) = d(x) * (d(x) * f(x))$
$d(x) \leq f(x)$

Hence, $d_f(x) \leq f(x)$ for all $x \in X$.

3.1.4. Proposition 4

Let $(X; *, 0)$ be a BCH-algebra, d is a mapping from X to itself.

1. Let d be (r, l)-(f, g)-derivation in X. If $f(x) * d(x) = 0$ for all $x \in X$, then d regular.
2. Let d be (l, r)-(f, g)-derivation in X. If $d(x) * f(x) = 0$ for all $x \in X$, then d regular.

Proof,

1. Let d is (r, l)-(f, g)-derivation in X. By axiom (BCH1) and (BCH7), for all $x \in X$ obtained:

$d(0) = d(x * x)$
$d(0) = (f(x) * d(x)) \wedge (d(x) * g(x))$
$d(0) = 0 \wedge (d(x) * g(x))$
$d(0) = \big(d(x) * g(x)\big) * ((d(x) * g(x)) * 0)$
$d(0) = \big(d(x) * g(x)\big) * (d(x) * g(x))$
$d(0) = 0$

Hence, $d(0) = 0$ then it is proved that d regular.

2. Let d is (l, r)-(f, g)-derivation in X. By axiom (BCH1) and (BCH7), for all $x \in X$ obtained:

$d(0) = d(x * x)$
$d(0) = (d(x) * f(x)) \wedge (g(x) * d(x))$
$d(0) = 0 \wedge (g(x) * d(x))$
$d(0) = \big(g(x) * d(x)\big) * ((g(x) * d(x)) * 0)$
$d(0) = \big(g(x) * d(x)\big) * (g(x) * d(x))$
$d(0) = 0$

Hence, $d(0) = 0$ then it is proved that d regular.

4. CONCLUSION

This article discussed the definition of (l,r) and (r,l)-(f,g)-derivation in BCH-algebra and investigate its proporties. In general, the properties of (l,r) and (r,l)-(f,g)-derivation in BCH-algebra obtained for d which satisfies the regular properties that is when $d(0)=0$.

REFERENCES

[1] Hu, Q. P. (1983). On BCH-algebras. *Mathematics Seminar Notes*, **11**, 313–320.

[2] Chaudhry, M. A. & Fakhar-Ud-Din, H. (2001). On some classes of BCH-algebras. *International Journal of Mathematics and Mathematical Sciences*, **25**(3), 205–211.

[3] Hu, Q. P. (1985). On proper BCH-algebras. *Math. Japonica*, **30**, 659–661.

[4] Kim, K. H. & Lee, Y. H. (2017). On left derivations of BCH-algebras. *The Korean Journal of Mathematics*, **25**(2), 163–179.

[5] Aprijon, A. (2021). Annual premium of life in insurance with uniform assumptions. *Science, Technology & Communication Journal*, **1**(2), 67–73.

[6] Lee, J. G., Hur, K., & Jun, Y. B. (2020). Multi-folded N-structures with finite degree and its application in BCH-algebras. *International Journal of Computational Intelligence Systems*.

[7] Rahmawati, R., Rahma, A. N., & Septia, W. (2021). Prediction of rupiah exchange rate against Australian Dollar using the Chen fuzzy time series method. *Science, Technology & Communication Journal*, **1**(2), 74–81.

[8] Chandramouleeswaran, M., Muralikrishna, P., Sujatha, K., & Sabarinathan, S. (2017). A note on Z-algebra. *Italian Journal of Pure and Applied Mathematics*, **38**, 707–714.

[9] El-Gendy, O. R. (2019). On BOI-Algebras. *International Journal of Mathematics and Computer Applications Research (IJMCAR)*, **9**(2), 13–28.

[10] Jun, Y. B. & Xin, X. L. (2004). On derivations of BCI-algebras. *Information Sciences*, **159**(3-4), 167–176.

[11] Zhan, J. & Liu, Y. L. (2005). On f-derivations of BCI-algebras. *International Journal of Mathematics and Mathematical Sciences*, **2005**(11), 1675–1684.

[12] Abujabal, H. A. & Al-Shehri, N. O. (2007). On left derivations of BCI-algebras. *Soochow Journal of Mathematics*, **33**(3), 435.

[13] Muhiuddin, G. & Al-Roqi, A. M. (2012). On-derivations in BCI-algebras. *Discrete Dynamics in Nature and Society*, 2012.

[14] Chen, L. (2011). Endomorphisms and order relations of sub-BCI-algebras. *2011 Fourth International Conference on Information and Computing*, 233–236.

[15] Nisar, F. (2009). Characterization of f-derivations of a BCI-algebra. *East Asian mathematical journal*, **25**(1), 69–87.

[16] Muhiuddin, G. (2016). Regularity of generalized derivations in BCI-algebras. *Communications of the Korean Mathematical Society*, **31**(2), 229–235.

[17] Iampan, A. (2016). Derivations of UP-algebras by means of UP-endomorphisms. *Algebraic Structures and Their Applications*, **3**(2), 1–20.

[18] Jana, C., Senapati, T., & Pal, M. (2017). t-derivations on complicated subtraction algebras. *Journal of Discrete Mathematical Sciences and Cryptography*, **20**(8), 1583–1595.

[19] Corina Ciungu, L. (2019). Pseudo-BCI algebras with derivations. *Journal of Logic and Computation*, **29**(8), 1367–1389.

[20] Muhiuddin, G. (2019). On (σ, τ)-Derivations of BCI-Algebras. *Southeast Asian Bulletin of Mathematics*, **43**(5).

[21] Al-Omary, R. M. (2019). On (α, β)(α, β)-derivations in d-algebras. *Bollettino dell'Unione Matematica Italiana*, **12**, 549–556.

[22] Alshehri, N. (2010). Derivations of B-algebras. *Science*, **22**(1).

[23] Ardekani, L. K. & Davvaz, B. (2014). On (f,g)-derivations of B-algebras. *Mat. Vesn*, **66**, 125–132.

[24] Bawazeer, S. M. & Bashammakh, S. A. (2015). Derivation of BCH-algebras and its fixed set. *Far East Journal of Mathematical Sciences*, **96**(8), 1029.

[25] Dar, K. H. & Akram, M. (2006). On endomorphisms of BCH-algebras. *Annals of the University of Craiova-Mathematics and Computer Science Series*, **33**, 227–234.

[26] Saeed, H. M. A. (2011). On closed BCH-algebra with respect to an element of a BCH-algebra. *Journal of Kufa for Mathematics and Computer*, **1**(4), 5–13.
[27] Mancao, J. & Canoy, S. (2020). Another look at topological BCH-algebras. *European Journal of Pure and Applied Mathematics*, **13**(4), 730–738.
[28] Thiruveni, V. & Jansi, M. (2018). Ideals and separation axioms in LSTBCH-algebras. *International Journal of Mathematics Trends and Technology*, **60**(2), 86–89.