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On left f_q-derivations of *B*-algebras

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ABSTRACT

In this paper, we introduce the notion of left f_q-derivation of *B*-algebra and investigate some related properties. Among them are properties of left f_q-derivation d_0^f of *B*-algebra $(X;*,0)$ and given properties of $d_q^f(x)$. Then, we discuss the properties of the regularleft f_q-derivation on *B*-algebras and composition properties of f_q-derivation on particular *B*-algebra, namely on *BM*-algebra.

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1. INTRODUCTION

Neggers and Kim (2002) introduce the notion of *B*-algebra [1], which is a non-empty set X with a constant 0 and a binary operation "$*$" donated by $(X;*,0)$, satisfying the following axioms $(B1)$ $x*x=0$, $(B2)$ $x*0=x$, and $(B3)$ $(x*y)*z=x*(z*(0*y))$ for all $x,y,z\in X$. Then, Kim and Kim (2008) introduce the notion of *BG*-algebra [2], which is the generalization of *B*-algebra satisfying the following axioms (*B*1), (*B*2), and (BG) $(x*y)*(0*y)=x$ for all $x,y\in X$. Kim and Park (2005) introduce 0-commutative *B*-algebra satisfying the following axioms $x*(0*y)=y*(0*x)$ for all $x,y\in X$ [3]. Kim and Kim (2006) also introduce *BM*-algebra [4], which is a specialization of *B*-algebra, satisfying the following axioms (*B*2) and (*A*2) $(z*x)*(z*x)=y*x$ for all $x,y,z\in X$. The relationship between *B*-algebra and *BM*-algebra is that every *BM*-algebra is *B*-algebra and every 0-commutative *B*-algebra is *BM*-algebra [5-8].

The first time, the notion of derivation is discussed in ring and near ring. In thedevelopment of abstract algebra, the notion of derivation is also discussed in otheralgebraic structures [9-13]. Abujabal and Al-Shehri (2007) introduce the left derivation on *BCI*-algebra [14], and then Al-Shehri (2010) introduces the derivation of *B*-algebra [15]. The results define a left-right or (l,r)-derivation, a right-left or (r,l)-derivation, and a regular in *B*-algebra. Then, also obtained the properties of the derivation on *B*-algebra. The concept of f_q-derivation is another type of derivation, as discussed by Al-Kadi (2016) regarding f_q-derivation on *G*-algebra [16]. Furthermore, Muangkarn et al. (2021) discussed the concept of f_q-derivation on *B*-algebra by defining a mapping involving endomorphisms [17]. However, the article has not discussed the properties of left f_q-derivation of *B*-algebra.

This article defines the concept of left f_q-derivation on *B*-algebra so that its properties are obtained. Then, we discuss the properties of theregular left f_q-derivation and the f_q-derivation composition properties of *BM*-algebra.

2. PRELIMINERIES

In this section, some definitions are needed to construct the research's primary results, with definitions and theories about *B*-algebra and *BM*-algebra. Then, given the left derivation concept of *BCI*-algebra and f_q-derivation of *B*-algebra, which have been discussed in [1, 3, 14, 15, 17-19, 20-25].

Definition 2.1. A *B*-algebra is a non-empty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms [1]:

(*B1*) $x * x = 0$,
(*B2*) $x * 0 = x$,
(*B3*) $(x * y) * z = x * (z * (0 * y))$,

for all $x, y \in X$.

Example 2.1. Let *X* = {0, 1, 2, 3, 4, 5} be a set with Cayley's table as seen in Table 1.

Table 1. Cayley's table for $(X;*,0)$.

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

It can be seen in Table 1 that the main diagonal is 0, so it applies $x * x = 0$ (*B1*) and the value in the second column represents the result of the binary operation, which is itself so that it satisfies $x * 0 = x$ (*B2*). Then, suppose $x, y \in X$, from Table 1 it can be proved that $(x * y) * z = x * (z * (0 * y))$ (*B3*). So, $(X;*,0)$ is a *B*-algebra.

Lemma 2.2. If $(X;*,0)$ is a *B*-aljabar [1], then

(i) $0 * (0 * x) = x$,
(ii) $(x * y) * (0 * y) = x$,
(iii) $y * z = y * (0 * (0 * z))$,
(iv) $x * (y * z) = (x * (0 * z)) * y$,
(v) $x * z = y * z$ implies $x = y$,
(vi) $x * y = 0$ implies $x = y$, for all $x, y, z \in X$.

Proof : Lemma 2.2 has been proved in [1].

Definition 2.3. A *B*-algebra $(X;*,0)$ is a 0-commutative *B*-algebra if it satisfies $x * (0 * y) = y * (0 * x)$ for all $x, y, z \in X$ [3].

Definition 2.4. A *BM*-algebra is a non-empty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms [4]:

(*A1*) $0 * x = x$,
(*A2*) $(z * x) * (z * y) = y * x$, for all $x, y, z \in X$.

Example 2.2. Let *X* = {0, 1, 2} be a set with Cayley's table as seen in Table 2.

It can be seen in Table 2 the value in the second column represents the result of the binary operation, which is itself so that it satisfies $x * 0 = x$ (*A1*). Then, suppose $x, y, z \in X$, from Table 2 it can be proved that $(z * x) * (z * y) = y * x$ (*A2*). So, $(X;*,0)$ is a *BM*-algebra.

Tabel 2. Tabel Cayley for $(X;*,0)$

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Lemma 2.5. If $(X;*,0)$ is a *BM*-aljabar [4], then

(i) $x * x = 0$,
(ii) $0 * (0 * x) = x$,
(iii) $0 * (x * y) = y * x$,
(iv) $(x * z) * (y * z) = x * y$,
(v) $x * y = 0$ if only if $y * x = 0$,

for all $x, y, z \in X$.

Proof. Lemma 2.5 has been proved in [4].

Theorem 2.6 Every *BM*-algebra is a *B*-algebra [4].

Proof. Theorem 2.6 has been proved in [4].

The converse of Theorem 2.6 does not hold in general. As in example 2.1 $(X;*,0)$ is a *B*-algebra but not *BM*-algebra since (5∗1)∗(5∗4) = 4≠5 = 4∗1.

Theorem 2.7 If $(X;*,0)$ is a *BM*-aljabar [4], then $(x * y) * z = (x * z) * y$ for all $x, y \in X$.

Proof. Theorem 2.7 has been proved in [4].

Definition 2.8 A Coxeter algebra is a non-empty set X with a constant 0 and a binary operation "∗" satisfying the following axioms [9]:

(*C1*) $x * x = 0$,
(*C2*) $x * 0 = x$,
(*C3*) $(x * y) * z = x * (y * z)$,

for all $x, y, z \in X$.

Theorem 2.9 If $(X;*,0)$ is a *BM*-algebra with $0 * x = x$ for all $x \in X$ [4], then $(X;*,0)$ is Coxeter algebra.

Proof. Theorem 2.9 has been proved in [4].

Corollary 2.10 An algebra $(X;*,0)$ is a Coxeter algebra if and only if it is a *BM*-algebra with $0 * x = x$ for all $x \in X$ [4].

Proof. Corollary 2.10 has been proved in [4].

The concept of derivation on *B*-algebra has been discussed in [6]. Let $(X;*,0)$ is a *B*-algebra, then $x \wedge y = y * (y * x)$, for all $x, y \in X$.

Definition 2.11 Let $(X;*,0)$ be a *B*-algebra [6]. A mapping of d from X to itself is called $(l,)$-derivation of X if it satisfies $d(x * y) = (d(x) * y) \wedge (x * d(y))$ for all $x, y \in X$ and we say that d is a $(r,)$-derivation of X if it satisfies $d(x * y) = (x * d(y)) \wedge (d(x) * y)$ for all $x, y \in X$. Moreover, if d is both an $(l,)$-derivation and an $(r,)$-derivation, we say that d is a derivation of X.

Let $(X;*,0)$ is a *B*-algebra. A mapping of d from X to itself is called regular if it satisfies $d(0) = 0$.

Definition 2.12 Let $(X;*,0)$ be a *BCI*-algebra [5]. By a left derivation of X, we mean a self-map d of X satisfying $d(x*y)=(x*d(y))\wedge(y*d(x))$ for all $x,y\in X$.

A self-map $f$on a *B*-algebra $X=(X;*,0)$ is called an endomorphism if $f(x*y)=f(x)*f(y)$ for all $x,y\in X$. The self-map d_q^f on X is defined by $d_q^f(X)=f(x)*q$ for all $x,q\in X$.

Definition 2.13 Let $f$be an endomorphism of a *B*-algebra $X=(X;*,0)$ [8]. A self-map d_q^fon X is called:

1) An inside f_q-derivation of X if $d_q^f(x*y)=d_q^f(x)*f(y)$ for all $x,y\in X$.
2) An outsidef_q-derivation of X if $d_q^f(x*y)=f(x)*d_q^f(y)$ for all $x,y\in X$.

an f_q-derivation of X if it is both an outside and inside f_q-derivation of X.

3. RESULTS AND DISCUSSIONS

This section provides the study's preliminary results, namely defining left f_q-derivationon *B*-algebra using the same method as defining left derivation on *BCI*-algebra. Then, the properties are given by the left f_q-derivationon *B*-algebra and the properties of thef_q-derivation composition on *BM*-algebra.

Definition 3.1 Let $(X;*,0)$ be a *B*-algebra and f is endomorphism of X. A self-map d_q^f on X is called left f_q-derivation of X satisfying $d_q^f(x*y)=(f(x)*d_q^f(y))\wedge(f(y)*d_q^f(x))$ for all $x,y\in X$.

Example 3.1 Let $(\mathbb{Z};-,0)$ be *B*-algebra. We define the mapping of f and d_q^f of $\mathbb{Z}$ to itself with $f(x)=x$ and $d_q^f(x)=f(x)-q$ for all $x\in\mathbb{Z}$. It can easily be proven that f is an endomorphism of $\mathbb{Z}$. It will be checked whether d_q^f is left f_q-derivation of $\mathbb{Z}$. For all $x,y\in\mathbb{Z}$ is obtained $d_q^f(x-y)=f(x-y)-q=x-y-q$ and,

$$(f(x)-d_q^f(y))\wedge(f(y)-d_q^f(x))=(f(x)-(f(y)-q))\wedge(f(y)-(f(x)-q))$$
$$(f(x)-d_q^f(y))\wedge(f(y)-d_q^f(x))=(x-y-q)\wedge(y-x+q)$$
$$(f(x)-d_q^f(y))\wedge(f(y)-d_q^f(x))=(y-x+q)-[(y-x+q)-(x-y-q)]$$
$$(f(x)-d_q^f(y))\wedge(f(y)-d_q^f(x))=x-y-q$$

So that it satisfies $d_q^f(x-y)=(f(x)-d_q^f(y))\wedge(f(y)-d_q^f(x))$. Thus, it is proved that d_q^f is left f_q-derivation on $\mathbb{Z}$.

Let $(X;*,0)$ be a *B*-algebra. A mapping of d_q^f on X to itself is called regular if it satisfies $d_q^f(0)=0$.

Theorem 3.2. Let $(X;*,0)$ be a *B*-algebra and f is an endomorphism of X. If d_q^f is left f_q-derivation on X, then

(i) $d_q^f(0)=f(x)*d_q^f(x)$ for all $x\in X$,
(ii) d_0^f is regular.

Proof. Let $(X;*,0)$ be a *B*-algebra and f is an endomorphism of X.

(i) Since d_q^f is left f_q-derivation on X, by the axiom *B1* and *B2* we get:

$d_q^f(0) = d_q^f(x * x)$
$d_q^f(0) = (f(x) * d_q^f(x)) \wedge (f(x) * d_q^f(x))$
$d_q^f(0) = \left(f(x) * d_q^f(x)\right) * [\left(f(x) * d_q^f(x)\right) * (f(x) * d_q^f(x))]$
$d_q^f(0) = \left(f(x) * d_q^f(x)\right) * 0$
$d_q^f(0) = f(x) * d_q^f(x)$

Hence, it is obtained that $d_q^f(0) = f(x) * d_q^f(x)$ for all $x \in X$.

(ii) By (1) and axiom *B1* we have:

$d_0^f(0) = f(x) * d_0^f(x)$
$d_0^f(0) = f(x) * (f(x) * 0)$
$d_0^f(0) = f(x) * f(x)$
$d_0^f(0) = 0$

So, it is obtained that d_0^f is regular.

Theorem 3.3. Let $(X;*,0)$ be a *B*-algebra, f is an endomorphism of X and d_q^f is left f_q-derivation regular on *X*. d_q^f is the identity function if and only if f is the identity function.

Proof. Let d_q^f is left f_q-derivation regular on *X*. Since d_q^f is the identity function, then $d_q^f(x) = x$ for all $x \in X$. By theorem 3.2 (1), axiom *B1* and lemma 2.2 (v) we have:

$$d_q^f(0) = 0$$
$$f(x) * d_q^f(x) = 0$$
$$f(x) * x = x * x$$
$$f(x) = x$$

thus, it is proved that f is an identity function. Conversely, if f is an identity function, then $f(x) = x$ for all$x \in X$. By theorem 3.2 (1), axiom *B1* and lemma 2.2 (v), we have:

$$d_q^f(0) = 0$$
$$f(x) * d_q^f(x) = 0$$
$$x * d_q^f(x) = d_q^f(x) * d_q^f(x)$$
$$x = d_q^f(x)$$

so, it is proved that d_q^f is an identity function.

Theorem 3.4 Let $(X;*,0)$ be a *B*-algebra, f is an endomorphism of X and d_q^f is left f_q-derivation on *X*. d_q^fregular if and only if $f = d_q^f$.

Proof. Let d_q^f is regular on *X*. By theorem 3.2 (1), axiom *B1* and lemma 2.2 (v) for all $x \in X$ are obtained:

$$d_q^f(0) = 0$$
$$f(x) * d_q^f(x) = d_q^f(x) * d_q^f(x)$$
$$f(x) = d_q^f(x)$$

thus, it is proved that $f = d_q^f$. Conversely, suppose $f = d_q^f$. Based on theorem 3.2 (i) and the axiom *B1* is obtained:

$$\begin{aligned} d_q^f(0) &= f(x) * d_q^f(x) \\ &= f(x) * f(x) \\ d_q^f(0) &= 0. \end{aligned}$$

so, it is proved that $d_q^f$is regular on *X*.

BM-algebra is a particular form of *B*-algebra, so the definition of inside and outside f_q -derivation on *BM*-algebra is the same as on *B*-algebra. The concept of left f_q-derivation on *BM*-algebra will not be discussed further because on *BM*-algebra $(X;* ,0)$ it applies $x \wedge y = y * (y * x) = x$ for all $x, y \in X$. Therefore, the concept of left f_q-derivation on *BM*-algebra is the same as outside f_q-derivation on *BM*-algebra.

Definition 3.5 Let $(X;* ,0)$ be a *B*-algebra, f is an endomorphism of X, d_q^f and D_q^f are mappings from *X* to itself. $d_q^f \circ D_q^f : X \to X$ is defined as $d_q^f \circ D_q^f(x) = d_q^f(D_q^f(x))$ for all $x \in X$.

Here are given the properties derived from the concept of composition f_q-derivation on *BM*-algebra.

Theorem 3.6 If $(X;* ,0)$ be a *BM*-algebra and *f* is the identity endomorphism of *X*, then $d_0^f \circ D_0^f$ is f_q-derivation of *X*.

Proof. We show that $d_0^f \circ D_0^f$ is inside f_q-derivation as well outside f_q-derivation on *X*. By axioms (A1) on *BM*-algebra for each $x \in X$ obtained:

$$\begin{aligned} (d_0^f \circ D_0^f)(x) &= d_0^f\left(D_0^f(x)\right) \\ (d_0^f \circ D_0^f)(x) &= d_0^f(f(x) * 0) \\ (d_0^f \circ D_0^f)(x) &= d_0^f(f(x)) \\ (d_0^f \circ D_0^f)(x) &= d_0^f(x) \\ (d_0^f \circ D_0^f)(x) &= f(x) * 0 \\ (d_0^f \circ D_0^f)(x) &= f(x) \end{aligned}$$

so that

$$\begin{aligned} \left(d_0^f \circ D_0^f\right)(x * y) &= f(x * y) * 0 \\ \left(d_0^f \circ D_0^f\right)(x * y) &= f(x * y) \\ \left(d_0^f \circ D_0^f\right)(x * y) &= f(x) * f(y) \\ \left(d_0^f \circ D_0^f\right)(x * y) &= (d_0^f \circ D_0^f)(x) * f(y) \end{aligned}$$

for all $x, y \in X$. Thus, it is proved that $d_0^f \circ D_0^f$is the inside f_q-derivation on *X*. Then, also from the axiom (*A1*) on *BM*-algebra, we get:

$$\begin{aligned} \left(d_0^f \circ D_0^f\right)(x * y) &= f(x * y) * 0 \\ \left(d_0^f \circ D_0^f\right)(x * y) &= f(x * y) \\ \left(d_0^f \circ D_0^f\right)(x * y) &= f(x) * f(y) \\ (d_0^f \circ D_0^f)(x * y) &= f(x) * (d_0^f \circ D_0^f)(y) \end{aligned}$$

for all $x, y \in X$. Thus, it is proved that $d_0^f \circ D_0^f$ is outside f_q-derivation on *X*. It is therefore proven that $d_0^f \circ D_0^f$ is f_q-derivation on *X*.

Theorem 3.7 Let $(X;*,0)$ be a *BM*-algebra and *f* is the identity endomorphism of *X*.

(i) If d_q^f and D_q^f are inside f_q-derivation on *X*, then $d_q^f \circ D_q^f$ is inside f_q-derivation on *X*
(ii) If d_q^f and D_q^f are outside f_q-derivation on *X*, then $d_q^f \circ D_q^f$ is outside f_q-derivation on *X*

Proof. Let $(X;*,0)$ be a *BM*-algebra and *f* is the identity endomorphism of *X*

(i) Since d_q^f and D_q^f are inside f_q-derivation on *X*, we have:

$$\begin{aligned}(d_q^f \circ D_q^f)(x*y) &= d_q^f(D_q^f(x*y))\\ &= d_q^f(D_q^f(x)*f(y))\\ &= d_q^f(D_q^f(x))*f(f(y))\\ (d_q^f \circ D_q^f)(x*y) &= (d_q^f \circ D_q^f)(x)*f(y),\end{aligned}$$

for all $x, y \in$ X. Thus, it is proved that $d_0^f \circ D_0^f$ is inside f_q-derivation of *X*.

(ii) Since d_q^f and D_q^f are outside f_q-derivation on *X*, we have:

$$\begin{aligned}(d_q^f \circ D_q^f)(x*y) &= d_q^f(D_q^f(x*y))\\ &= d_q^f(f(x)*D_q^f(y))\\ &= f(f(x))*d_q^f(D_q^f(y))\\ (d_q^f \circ D_q^f)(x*y) &= f(x)*(d_q^f \circ D_q^f)(y),\end{aligned}$$

for all $x, y \in$ X. Thus, it is proved that $d_0^f \circ D_0^f$ is outside f_q-derivation of *X*.

Corollary 3.8 Let $(X;*,0)$ be a *BM*-algebra, and *f* is the identity endomorphism of *X*. If d_q^f and D_q^f are f_q-derivation on *X*, then $d_q^f \circ D_q^f$ is f_q-derivation on *X*.

Proof. The corollary of 3.8 is immediately evident based on theorem 3.7 (i) and (ii).

Theorem 3.9 Let $(X;*,0)$ be a *BM*-algebra satisfying $x*y=y*x$ for all $x,y \in X$. *f* is the identity endomorphism of *X*, d_q^f and D_q^f are f_q-derivation on *X*. If $d_q^f \circ f = f \circ d_q^f$ and $D_q^f \circ f = f \circ D_q^f$, then $d_q^f \circ D_q^f = D_q^f \circ d_q^f$.

Proof. Since d_q^f and D_q^f are f_q-derivation on *X* and $d_q^f \circ f = f \circ d_q^f$, $D_q^f \circ f = f \circ D_q^f$ then,

$$\begin{aligned}(d_q^f \circ D_q^f)(x*y) &= d_q^f(D_q^f(x*y))\\ (d_q^f \circ D_q^f)(x*y) &= d_q^f(D_q^f(x)*f(y))\\ (d_q^f \circ D_q^f)(x*y) &= d_q^f(f(y)*D_q^f(x))\\ (d_q^f \circ D_q^f)(x*y) &= d_q^f(f(y))*f(D_q^f(x))\\ (d_q^f \circ D_q^f)(x*y) &= (d_q^f \circ f)(y)*(f \circ D_q^f)(x)\\ (d_q^f \circ D_q^f)(x*y) &= (f \circ d_q^f)(y)*(D_q^f \circ f)(x)\end{aligned}$$

for all $x, y \in$ X. On the other is obtained:

$$\begin{aligned}(D_q^f \circ d_q^f)(x*y) &= D_q^f(d_q^f(x*y))\\ (D_q^f \circ d_q^f)(x*y) &= D_q^f(f(x)*d_q^f(y))\end{aligned}$$

$$\left(D_q^f \circ d_q^f\right)(x * y) = D_q^f(f(x)) * f(d_q^f(y))$$
$$\left(D_q^f \circ d_q^f\right)(x * y) = (D_q^f \circ f)(x) * (f \circ d_q^f)(y)$$
$$\left(D_q^f \circ d_q^f\right)(x * y) = (f \circ d_q^f)(y) * (D_q^f \circ f)(x)$$

for all $x, y \in$ X. So, it is proved that $d_q^f \circ D_q^f = D_q^f \circ d_q^f$.

4. CONCLUSION

In this article, it can be concluded that the properties of left f_q-derivation are obtained in *B*-algebra. However, most of the properties are accepted for a regular leftf_q-derivation. Then, the concept of left f_q-derivation on *BM*-algebra is not discussed in depth because it is equivalent to outside f_q-derivation on *BM*-algebra. Thecomposition of f_q-derivation properties on *BM*-algebra is only obtained if *f* is an identity endomorphism on *BM*-algebra.

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