

# On Left $f_q$ -Derivations of $B$ -Algebras

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## ABSTRACT

In this paper, we introduce the notion of left  $f_q$ -derivation of  $B$ -algebra and investigate some related properties. Among them are properties of left  $f_q$ -derivation  $d_0^f$  of  $B$ -algebra  $(X; *, 0)$  and given properties of  $d_q^f(x)$ . Then, we discuss the properties of the regular left  $f_q$ -derivation on  $B$ -algebras and composition properties of  $f_q$ -derivation on particular  $B$ -algebra, namely on  $BM$ -algebra.

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## 1. INTRODUCTION

Negggers and Kim [1] introduce the notion of  $B$ -algebra, which is a non empty set  $X$  with a constant  $0$  and a binary operation “ $*$ ” donated by  $(X; *, 0)$ , satisfying the following axioms (B1)  $x * x = 0$ , (B2)  $x * 0 = x$ , and (B3)  $(x * y) * z = x * (z * (0 * y))$  for all  $x, y, z \in X$ . Then, Kim and Kim [2] introduce the notion of  $BG$ -algebra which is the generalization of  $B$ -algebra satisfying the following axioms (B1), (B2), and (BG)  $(x * y) * (0 * y) = x$  for all  $x, y \in X$ . Kim and Park [3] introduce 0-commutative  $B$ -algebra satisfying the following axioms  $x * (0 * y) = y * (0 * x)$  for all  $x, y \in X$ . Kim and Kim [4] also introduce  $BM$ -algebra, which is a specialization of  $B$ -algebra, satisfying the following axioms (B2) and (A2)  $(z * x) * (z * x) = y * x$  for all  $x, y, z \in X$ . The relationship between  $B$ -algebra and  $BM$ -algebra is that every  $BM$ -algebra is  $B$ -algebra and every 0-commutative  $B$ -algebra is  $BM$ -algebra.

The first time, the notion of derivation is discussed in ring and near ring. In the development of abstract algebra, the notion of derivation is also discussed in other algebraic structures. Abujabal and Al-Shehrie [5] introduce the left derivation on  $BCI$ -algebra, and then Al-Shehrie [6] introduces the derivation of  $B$ -algebra. The results define a left-right or  $(l, r)$ -derivation, a right-left or  $(r, l)$ -derivation, and a regular in  $B$ -algebra. Then, also obtained the properties of the derivation on  $B$ -algebra. The concept of  $f_q$ -derivation is another type of derivation, as discussed by Al-Kadi [7] regarding  $f_q$ -derivation on  $G$ -algebra. Furthermore, Muangkarn et al. [8] discussed the concept of  $f_q$ -derivation on  $B$ -algebra by defining a mapping involving endomorphisms. However, the article has not discussed the properties of left  $f_q$ -derivation of  $B$ -algebra.

This article defines the concept of left  $f_q$ -derivation on  $B$ -algebra so that its properties are obtained. Then, we discuss the properties of the regular left  $f_q$ -derivation and the  $f_q$ -derivation composition properties of  $BM$ -algebra.

## 2. PRELIMINERIES

In this section, some definitions are needed to construct the research's primary results, with definitions and theories about  $B$ -algebra and  $BM$ -algebra. Then, given the left derivation concept of  $BCI$ -algebra and  $f_q$ -derivation of  $B$ -algebra, which have been discussed in [1, 3-6, 8-12].

**Definition 2.1.** A  $B$ -algebra is a non-empty set  $X$  with a constant  $0$  and a binary operation “ $*$ ” satisfying the following axioms [1]:

- (B1)  $x * x = 0$ ,  
 (B2)  $x * 0 = x$ ,  
 (B3)  $(x * y) * z = x * (z * (0 * y))$ ,  
 for all  $x, y \in X$ .

**Example 2.1.** Let  $X = \{0, 1, 2, 3, 4, 5\}$  be a set with Cayley’s table as seen in Table 1.

Table 1. Cayley’s table for  $(X; *, 0)$

*	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

It can be seen in Table 1 that the main diagonal is 0, so it applies  $x * x = 0$  (B1) and the value in the second column represents the result of the binary operation, which is itself so that it satisfies  $x * 0 = x$  (B2). Then, suppose  $x, y \in X$ , from Table 1 it can be proved that  $(x * y) * z = x * (z * (0 * y))$  (B3). So,  $(X; *, 0)$  is a  $B$ -algebra.

**Lemma 2.2.** If  $(X; *, 0)$  is a  $B$ -aljabar [1], then

- (i)  $0 * (0 * x) = x$ ,  
 (ii)  $(x * y) * (0 * y) = x$ ,  
 (iii)  $y * z = y * (0 * (0 * z))$ ,  
 (iv)  $x * (y * z) = (x * (0 * z)) * y$ ,  
 (v)  $x * z = y * z$  implies  $x = y$ ,  
 (vi)  $x * y = 0$  implies  $x = y$ , for all  $x, y, z \in X$ .

**Proof :** Lemma 2.2 has been proved in [1].

**Definition 2.3.** A  $B$ -algebra  $(X; *, 0)$  is a 0-commutative  $B$ -algebra if it satisfies  $x * (0 * y) = y * (0 * x)$  for all  $x, y, z \in X$  [3].

**Definition 2.4.** A  $BM$ -algebra is a non-empty set  $X$  with a constant  $0$  and a binary operation “ $*$ ” satisfying the following axioms [4]:

- (A1)  $0 * x = x$ ,  
 (A2)  $(z * x) * (z * y) = y * x$ , for all  $x, y, z \in X$ .

**Example 2.2.** Let  $X = \{0, 1, 2\}$  be a set with Cayley’s table as seen in Table 2.

Tabel 2. Tabel Cayley for  $(X; *, 0)$

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

It can be seen in Table 2 the value in the second column represents the result of the binary operation, which is itself so that it satisfies  $x * 0 = x$  (A1). Then, suppose  $x, y, z \in X$ , from Table 2 it can be proved that  $(z * x) * (z * y) = y * x$  (A2). So,  $(X; *, 0)$  is a *BM*-algebra.

**Lemma 2.5.** If  $(X; *, 0)$  is a *BM*-aljabar [4], then

- (i)  $x * x = 0$ ,
  - (ii)  $0 * (0 * x) = x$ ,
  - (iii)  $0 * (x * y) = y * x$ ,
  - (iv)  $(x * z) * (y * z) = x * y$ ,
  - (v)  $x * y = 0$  if only if  $y * x = 0$ ,
- for all  $x, y, z \in X$ .

**Proof.** Lemma 2.5 has been proved in [4].

**Theorem 2.6** Every *BM*-algebra is a *B*-algebra [4].

**Proof.** Theorem 2.6 has been proved in [4].

The converse of Theorem 2.6 does not hold in general. As in example 2.1  $(X; *, 0)$  is a *B*-algebra but not *BM*-algebra since  $(5*1)*(5*4) = 4 \neq 5 = 4*1$ .

**Theorem 2.7** If  $(X; *, 0)$  is a *BM*-aljabar [4], then  $(x * y) * z = (x * z) * y$  for all  $x, y \in X$ .

**Proof.** Theorem 2.7 has been proved in [4].

**Definition 2.8** A Coxeter algebra is a non-empty set  $X$  with a constant  $0$  and a binary operation “ $*$ ” satisfying the following axioms [9]:

- (C1)  $x * x = 0$ ,
  - (C2)  $x * 0 = x$ ,
  - (C3)  $(x * y) * z = x * (y * z)$ ,
- for all  $x, y, z \in X$ .

**Theorem 2.9** If  $(X; *, 0)$  is a *BM*-algebra with  $0 * x = x$  for all  $x \in X$  [4], then  $(X; *, 0)$  is Coxeter algebra.

**Proof.** Theorem 2.9 has been proved in [4].

**Corollary 2.10** An algebra  $(X; *, 0)$  is a Coxeter algebra if and only if it is a *BM*-algebra with  $0 * x = x$  for all  $x \in X$  [4].

**Proof.** Corollary 2.10 has been proved in [4].

The concept of derivation on *B*-algebra has been discussed in [6]. Let  $(X; *, 0)$  is a *B*-algebra, then  $x \wedge y = y * (y * x)$ , for all  $x, y \in X$ .

**Definition 2.11** Let  $(X; *, 0)$  be a *B*-algebra [6]. A mapping of  $d$  from  $X$  to itself is called  $(l,r)$ -derivation of  $X$  if it satisfies  $d(x * y) = (d(x) * y) \wedge (x * d(y))$  for all  $x, y \in X$  and we say that  $d$  is a  $(r,l)$ -derivation of  $X$  if it satisfies  $d(x * y) = (x * d(y)) \wedge (d(x) * y)$  for all  $x, y \in X$ . Moreover, if  $d$  is both an  $(l,r)$ -derivation and an  $(r,l)$ -derivation, we say that  $d$  is a derivation of  $X$ .

Let  $(X; *, 0)$  is a *B*-algebra. A mapping of  $d$  from  $X$  to itself is called regular if it satisfies  $d(0) = 0$ .

**Definition 2.12** Let  $(X; *, 0)$  be a *BCI*-algebra [5]. By a left derivation of  $X$ , we mean a self-map  $d$  of  $X$  satisfying  $d(x * y) = (x * d(y)) \wedge (y * d(x))$  for all  $x, y \in X$ .

A self-map  $f$  on a *B*-algebra  $X = (X; *, 0)$  is called an endomorphism if  $f(x * y) = f(x) * f(y)$  for all  $x, y \in X$ . The self-map  $d_q^f$  on  $X$  is defined by  $d_q^f(X) = f(x) * q$  for all  $x, q \in X$ .

**Definition 2.13** Let  $f$  be an endomorphism of a  $B$ -algebra  $X = (X; *, 0)$  [8]. A self-map  $d_q^f$  on  $X$  is called

- 1) An inside  $f_q$ -derivation of  $X$  if  $d_q^f(x * y) = d_q^f(x) * f(y)$  for all  $x, y \in X$ .
- 2) An outside  $f_q$ -derivation of  $X$  if  $d_q^f(x * y) = f(x) * d_q^f(y)$  for all  $x, y \in X$ .
- 3) An  $f_q$ -derivation of  $X$  if it is both an outside and inside  $f_q$ -derivation of  $X$ .

### 3. RESULTS AND DISCUSSIONS

This section provides the study's preliminary results, namely defining left  $f_q$ -derivation on  $B$ -algebra using the same method as defining left derivation on  $BCI$ -algebra. Then, the properties are given by the left  $f_q$ -derivation on  $B$ -algebra and the properties of the  $f_q$ -derivation composition on  $BM$ -algebra.

**Definition 3.1** Let  $(X; *, 0)$  be a  $B$ -algebra and  $f$  is endomorphism of  $X$ . A self-map  $d_q^f$  on  $X$  is called left  $f_q$ -derivation of  $X$  satisfying  $d_q^f(x * y) = (f(x) * d_q^f(y)) \wedge (f(y) * d_q^f(x))$  for all  $x, y \in X$ .

**Example 3.1** Let  $(\mathbb{Z}; -, 0)$  be  $B$ -algebra. We define the mapping of  $f$  and  $d_q^f$  of  $\mathbb{Z}$  to itself with  $f(x) = x$  and  $d_q^f(x) = f(x) - q$  for all  $x \in \mathbb{Z}$ . It can easily be proven that  $f$  is an endomorphism of  $\mathbb{Z}$ . It will be checked whether  $d_q^f$  is left  $f_q$ -derivation of  $\mathbb{Z}$ . For all  $x, y \in \mathbb{Z}$  is obtained  $d_q^f(x - y) = f(x - y) - q = x - y - q$  and,

$$\begin{aligned} (f(x) - d_q^f(y)) \wedge (f(y) - d_q^f(x)) &= (f(x) - (f(y) - q)) \wedge (f(y) - (f(x) - q)) \\ (f(x) - d_q^f(y)) \wedge (f(y) - d_q^f(x)) &= (x - y - q) \wedge (y - x + q) \\ (f(x) - d_q^f(y)) \wedge (f(y) - d_q^f(x)) &= (y - x + q) - [(y - x + q) - (x - y - q)] \\ (f(x) - d_q^f(y)) \wedge (f(y) - d_q^f(x)) &= x - y - q \end{aligned}$$

So that it satisfies  $d_q^f(x - y) = (f(x) - d_q^f(y)) \wedge (f(y) - d_q^f(x))$ . Thus, it is proved that  $d_q^f$  is left  $f_q$ -derivation on  $\mathbb{Z}$ .

Let  $(X; *, 0)$  be a  $B$ -algebra. A mapping of  $d_q^f$  on  $X$  to itself is called regular if it satisfies  $d_q^f(0) = 0$ .

**Theorem 3.2.** Let  $(X; *, 0)$  be a  $B$ -algebra and  $f$  is an endomorphism of  $X$ . If  $d_q^f$  is left  $f_q$ -derivation on  $X$ , then

- (i)  $d_q^f(0) = f(x) * d_q^f(x)$  for all  $x \in X$ ,
- (ii)  $d_q^f$  is regular.

**Proof.** Let  $(X; *, 0)$  be a  $B$ -algebra and  $f$  is an endomorphism of  $X$ .

- (i) Since  $d_q^f$  is left  $f_q$ -derivation on  $X$ , by the axiom  $B1$  and  $B2$  we get:

$$\begin{aligned} d_q^f(0) &= d_q^f(x * x) \\ d_q^f(0) &= (f(x) * d_q^f(x)) \wedge (f(x) * d_q^f(x)) \\ d_q^f(0) &= (f(x) * d_q^f(x)) * [(f(x) * d_q^f(x)) * (f(x) * d_q^f(x))] \\ d_q^f(0) &= (f(x) * d_q^f(x)) * 0 \\ d_q^f(0) &= f(x) * d_q^f(x) \end{aligned}$$

Hence, it is obtained that  $d_q^f(0) = f(x) * d_q^f(x)$  for all  $x \in X$ .

(ii) By (1) and axiom *BI* we have:

$$\begin{aligned}d_0^f(0) &= f(x) * d_0^f(x) \\d_0^f(0) &= f(x) * (f(x) * 0) \\d_0^f(0) &= f(x) * f(x) \\d_0^f(0) &= 0\end{aligned}$$

So, it is obtained that  $d_0^f$  is regular.

**Theorem 3.3.** Let  $(X;*,0)$  be a *B*-algebra,  $f$  is an endomorphism of  $X$  and  $d_q^f$  is left  $f_q$ -derivation regular on  $X$ .  $d_q^f$  is the identity function if and only if  $f$  is the identity function.

**Proof.** Let  $d_q^f$  is left  $f_q$ -derivation regular on  $X$ . Since  $d_q^f$  is the identity function, then  $d_q^f(x) = x$  for all  $x \in X$ . By theorem 3.2 (1), axiom *BI* and lemma 2.2 (v) we have:

$$\begin{aligned}d_q^f(0) &= 0 \\f(x) * d_q^f(x) &= 0 \\f(x) * x &= x * x \\f(x) &= x\end{aligned}$$

thus, it is proved that  $f$  is an identity function. Conversely, if  $f$  is an identity function, then  $f(x) = x$  for all  $x \in X$ . By theorem 3.2 (1), axiom *BI* and lemma 2.2 (v), we have:

$$\begin{aligned}d_q^f(0) &= 0 \\f(x) * d_q^f(x) &= 0 \\x * d_q^f(x) &= d_q^f(x) * d_q^f(x) \\x &= d_q^f(x)\end{aligned}$$

so, it is proved that  $d_q^f$  is an identity function.

**Theorem 3.4** Let  $(X;*,0)$  be a *B*-algebra,  $f$  is an endomorphism of  $X$  and  $d_q^f$  is left  $f_q$ -derivation on  $X$ .  $d_q^f$  regular if and only if  $f = d_q^f$ .

**Proof.** Let  $d_q^f$  is regular on  $X$ . By theorem 3.2 (1), axiom *BI* and lemma 2.2 (v) for all  $x \in X$  are obtained:

$$\begin{aligned}d_q^f(0) &= 0 \\f(x) * d_q^f(x) &= d_q^f(x) * d_q^f(x) \\f(x) &= d_q^f(x)\end{aligned}$$

thus, it is proved that  $f = d_q^f$ . Conversely, suppose  $f = d_q^f$ . Based on theorem 3.2 (i) and the axiom *BI* is obtained:

$$\begin{aligned}d_q^f(0) &= f(x) * d_q^f(x) \\&= f(x) * f(x) \\d_q^f(0) &= 0.\end{aligned}$$

so, it is proved that  $d_q^f$  is regular on  $X$ .

$BM$ -algebra is a particular form of  $B$ -algebra, so the definition of inside and outside  $f_q$ -derivation on  $BM$ -algebra is the same as on  $B$ -algebra. The concept of left  $f_q$ -derivation on  $BM$ -algebra will not be discussed further because on  $BM$ -algebra  $(X; *, 0)$  it applies  $x \wedge y = y * (y * x) = x$  for all  $x, y \in X$ . Therefore, the concept of left  $f_q$ -derivation on  $BM$ -algebra is the same as outside  $f_q$ -derivation on  $BM$ -algebra.

**Definition 3.5** Let  $(X; *, 0)$  be a  $B$ -algebra,  $f$  is an endomorphism of  $X$ ,  $d_q^f$  and  $D_q^f$  are mappings from  $X$  to itself.  $d_q^f \circ D_q^f: X \rightarrow X$  is defined as  $d_q^f \circ D_q^f(x) = d_q^f(D_q^f(x))$  for all  $x \in X$ .

Here are given the properties derived from the concept of composition  $f_q$ -derivation on  $BM$ -algebra.

**Theorem 3.6** If  $(X; *, 0)$  be a  $BM$ -algebra and  $f$  is the identity endomorphism of  $X$ , then  $d_0^f \circ D_0^f$  is  $f_q$ -derivation of  $X$ .

**Proof.** We show that  $d_0^f \circ D_0^f$  is inside  $f_q$ -derivation as well outside  $f_q$ -derivation on  $X$ . By axioms (A1) on  $BM$ -algebra for each  $x \in X$  obtained:

$$\begin{aligned} (d_0^f \circ D_0^f)(x) &= d_0^f(D_0^f(x)) \\ (d_0^f \circ D_0^f)(x) &= d_0^f(f(x) * 0) \\ (d_0^f \circ D_0^f)(x) &= d_0^f(f(x)) \\ (d_0^f \circ D_0^f)(x) &= d_0^f(x) \\ (d_0^f \circ D_0^f)(x) &= f(x) * 0 \\ (d_0^f \circ D_0^f)(x) &= f(x) \end{aligned}$$

so that

$$\begin{aligned} (d_0^f \circ D_0^f)(x * y) &= f(x * y) * 0 \\ (d_0^f \circ D_0^f)(x * y) &= f(x * y) \\ (d_0^f \circ D_0^f)(x * y) &= f(x) * f(y) \\ (d_0^f \circ D_0^f)(x * y) &= (d_0^f \circ D_0^f)(x) * f(y) \end{aligned}$$

for all  $x, y \in X$ . Thus, it is proved that  $d_0^f \circ D_0^f$  is the inside  $f_q$ -derivation on  $X$ . Then, also from the axiom (A1) on  $BM$ -algebra, we get:

$$\begin{aligned} (d_0^f \circ D_0^f)(x * y) &= f(x * y) * 0 \\ (d_0^f \circ D_0^f)(x * y) &= f(x * y) \\ (d_0^f \circ D_0^f)(x * y) &= f(x) * f(y) \\ (d_0^f \circ D_0^f)(x * y) &= f(x) * (d_0^f \circ D_0^f)(y) \end{aligned}$$

for all  $x, y \in X$ . Thus, it is proved that  $d_0^f \circ D_0^f$  is outside  $f_q$ -derivation on  $X$ . It is therefore proven that  $d_0^f \circ D_0^f$  is  $f_q$ -derivation on  $X$ .

**Theorem 3.7** Let  $(X; *, 0)$  be a  $BM$ -algebra and  $f$  is the identity endomorphism of  $X$ .

- (i) If  $d_q^f$  and  $D_q^f$  are inside  $f_q$ -derivation on  $X$ , then  $d_q^f \circ D_q^f$  is inside  $f_q$ -derivation on  $X$
- (ii) If  $d_q^f$  and  $D_q^f$  are outside  $f_q$ -derivation on  $X$ , then  $d_q^f \circ D_q^f$  is outside  $f_q$ -derivation on  $X$

**Proof.** Let  $(X; *, 0)$  be a  $BM$ -algebra and  $f$  is the identity endomorphism of  $X$

(i) Since  $d_q^f$  and  $D_q^f$  are inside  $f_q$ -derivation on  $X$ , we have:

$$\begin{aligned} (d_q^f \circ D_q^f)(x * y) &= d_q^f(D_q^f(x * y)) \\ &= d_q^f(D_q^f(x) * f(y)) \\ &= d_q^f(D_q^f(x)) * f(f(y)) \\ (d_q^f \circ D_q^f)(x * y) &= (d_q^f \circ D_q^f)(x) * f(y), \end{aligned}$$

for all  $x, y \in X$ . Thus, it is proved that  $d_0^f \circ D_0^f$  is inside  $f_q$ -derivation of  $X$ .

(ii) Since  $d_q^f$  and  $D_q^f$  are outside  $f_q$ -derivation on  $X$ , we have:

$$\begin{aligned} (d_q^f \circ D_q^f)(x * y) &= d_q^f(D_q^f(x * y)) \\ &= d_q^f(f(x) * D_q^f(y)) \\ &= f(f(x)) * d_q^f(D_q^f(y)) \\ (d_q^f \circ D_q^f)(x * y) &= f(x) * (d_q^f \circ D_q^f)(y), \end{aligned}$$

for all  $x, y \in X$ . Thus, it is proved that  $d_0^f \circ D_0^f$  is outside  $f_q$ -derivation of  $X$ .

**Corollary 3.8** Let  $(X; *, 0)$  be a  $BM$ -algebra, and  $f$  is the identity endomorphism of  $X$ . If  $d_q^f$  and  $D_q^f$  are  $f_q$ -derivation on  $X$ , then  $d_q^f \circ D_q^f$  is  $f_q$ -derivation on  $X$ .

**Proof.** The corollary of 3.8 is immediately evident based on theorem 3.7 (i) and (ii).

**Theorem 3.9** Let  $(X; *, 0)$  be a  $BM$ -algebra satisfying  $x * y = y * x$  for all  $x, y \in X$ .  $f$  is the identity endomorphism of  $X$ ,  $d_q^f$  and  $D_q^f$  are  $f_q$ -derivation on  $X$ . If  $d_q^f \circ f = f \circ d_q^f$  and  $D_q^f \circ f = f \circ D_q^f$ , then  $d_q^f \circ D_q^f = D_q^f \circ d_q^f$ .

**Proof.** Since  $d_q^f$  and  $D_q^f$  are  $f_q$ -derivation on  $X$  and  $d_q^f \circ f = f \circ d_q^f$ ,  $D_q^f \circ f = f \circ D_q^f$  then,

$$\begin{aligned} (d_q^f \circ D_q^f)(x * y) &= d_q^f(D_q^f(x * y)) \\ (d_q^f \circ D_q^f)(x * y) &= d_q^f(D_q^f(x) * f(y)) \\ (d_q^f \circ D_q^f)(x * y) &= d_q^f(f(y) * D_q^f(x)) \\ (d_q^f \circ D_q^f)(x * y) &= d_q^f(f(y)) * f(D_q^f(x)) \\ (d_q^f \circ D_q^f)(x * y) &= (d_q^f \circ f)(y) * (f \circ D_q^f)(x) \\ (d_q^f \circ D_q^f)(x * y) &= (f \circ d_q^f)(y) * (D_q^f \circ f)(x) \end{aligned}$$

for all  $x, y \in X$ . On the other is obtained:

$$\begin{aligned} (D_q^f \circ d_q^f)(x * y) &= D_q^f(d_q^f(x * y)) \\ (D_q^f \circ d_q^f)(x * y) &= D_q^f(f(x) * d_q^f(y)) \\ (D_q^f \circ d_q^f)(x * y) &= D_q^f(f(x)) * f(d_q^f(y)) \\ (D_q^f \circ d_q^f)(x * y) &= (D_q^f \circ f)(x) * (f \circ d_q^f)(y) \\ (D_q^f \circ d_q^f)(x * y) &= (f \circ d_q^f)(y) * (D_q^f \circ f)(x) \end{aligned}$$

for all  $x, y \in X$ . So, it is proved that  $d_q^f \circ D_q^f = D_q^f \circ d_q^f$ .

#### 4. CONCLUSION

In this article, it can be concluded that the properties of left  $f_q$ -derivation are obtained in  $B$ -algebra. However, most of the properties are accepted for a regular left  $f_q$ -derivation. Then, the concept of left  $f_q$ -derivation on  $BM$ -algebra is not discussed in depth because it is equivalent to outside  $f_q$ -derivation on  $BM$ -algebra. The composition of  $f_q$ -derivation properties on  $BM$ -algebra is only obtained if  $f$  is an identity endomorphism on  $BM$ -algebra.

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