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t-derivations in BP-algebras

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ABSTRACT

BP-algebras is a non-empty set $(X, *, 0)$ with the binary operation "$*$" satisfies the axioms (BP1) $x * x = 0$, (BP2) $x * (x * y) = y$, (BP3) $(x * z) * (y * z) = x * y$ for all $x, y, z \in X$. In this paper, we define the concept of (l, r) and (r, l) t-derivation in BP-algebra and investigate their properties. Based on the concepts of (l, r) and (r, l) t-derivation in BP-algebra, the properties of t-derivation in BP-algebra are constructed.

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1. INTRODUCTION

Algebraic structure is a fundamental topic, many researchers develop various types of algebraic structures are B-algebra, 0-commutative B-algebra, and BP-algebra. Neggers and Kim [1] introduced the concept of B-algebra, which is a non-empty set X with a binary operation "$*$" and a constant 0 denoted by $(X; *, 0)$, and satisfies the axioms $(B1)$ $x * x = 0$, $(B2)$ $x * 0 = x$, and (B3) $(x * y) * z = x * \left(z * (0 * y)\right)$ for all $x, y, z \in X$. Then, Kim and Park [2] discusses a special form of B-algebra called 0-commutative B-algebra, which is a non-empty set X with a binary operations $*$ and a constant 0 that satisfies $x * (0 * y) = y * (0 * x)$ for all $x, y \in X$.

Some important concepts in abstract algebra have been discussed, including the concept of derivation, which was first introduced in ring studies [3-9]. The concept of derivation is also discussed in other algebraic structures [10, 11], such as B-algebra. Besides in B-algebra, the concept of derivation is also found in BP-algebra and its properties which are discussed by Kandaraj and Devi [12, 13]. In this discussion, the properties of derivation and f-derivation in BP-algebra [10, 11, 14-17] are given which have differences with the properties of derivations in B-algebra [18-22]. In 2014, another concept of derivation was discussed, namely t-derivation in B-algebra [23], defining the concept of t-derivation in B-algebra resulting in a new type of derivation that is different from the concept of ordinary derivation [10, 11, 24]. The concept of t-derivation in B-algebra is obtained by defining $(l, r)t$-derivation, $(r, l)t$-derivation and t-regular in B-algebra. Besides that, the properties of the derivations in B-algebra are also obtained, which are expressed in the form of theorems.

Based on the description above, using the same way of constructing the concept of t-derivation in B-algebra by Rasoel et al. [23], namely by involving two mappings that can be constructed on the concept of t-derivation in BP-algebra and also determine the properties associated with the concept.

2. MATERIALS AND METHOD

In this section, the definitions of BP-algebra, B-algebra and 0-commutative B-algebra are defined, along with their properties.

2.1. Definition B-Algebra

B-algebra is a non-empty set X with the binary operation “$*$“ and a constant 0 which satisfies the following axioms [25]:

(B1) $x * x = 0$
(B2) $x * 0 = x$
(B3) $(x * y) * x = x * \big(z * (0 * y)\big)$

for all $x, y, z \in X$.

2.1.1. Proposition

If $(X,* ,0)$ is B-algebra, then:

i. $(x * y) * (0 * y) = x$,
ii. If $x * y = z * y$ then $x = z$,
iii. $x * (y * z) = \big(x * (0 * z)\big) * y$,
iv. If $x * y = 0$ then $x = y$,
v. $x = 0 * (0 * x)$,
vi. If $x * y = x * z$ then $y = z$,
vii. $0 * (x * y) = y * x$,
viii. $(x * y) * (z * y) = x * z$,

for all $x, y, z \in X$. The proof of this Proposition has been prove in [25].

2.2. Definition B-Algebra 0-Commutative

B-algebra $(X,*, 0)$ is said to be 0-commutative if $x * (0 * y) = y * (0 * x)$for all $x, y \in X$.

2.2.1. Proposition

If $(X,*, 0)$ is 0-commutative B-algebra, then:

i. $(x * z) * (y * w) = (w * z) * (y * x)$,
ii. $(x * z) * (y * z) = x * y$,
iii. $(z * y) * (z * x) = x * y$,
iv. $(x * z) * y = (0 * z) * (y * x)$,
v. $x * (y * z) = z * (y * x)$,
vi. $(x * y) * z = (x * z) * y$,
vii. $\big((x * y) * (x * z)\big) * (x * y) = 0$,
viii. $\big(x * (x * y)\big) * y = 0$,
ix. $x * (x * y) = y$,
x. If $x * y = x * z$, then $y = z$,

for all $w, x, y, z \in X$. The proof of this Proposition has been given in [25].

2.3. Definition BP-Algebra

BP-algebra is a non-empty set X with the binary operation "$*$" and a constant 0 which satisfies the following axiom [25]:

(BP1) $x * x = 0$,
(BP2) $x * (x * y) = y$,
(BP3) $(x * z) * (y * z) = x * y$,

for all $x, y, z \in X$.

Example, suppose $X \coloneqq \{0, a, b, c\}$ is a set with the Cayley table as follows:

Table 1. Cayley's table for $(X,*,0)$.

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

From Table 1, it can be proved that $(X,*,0)$ satisfies all axioms in BP-algebra. So that $(X,*,0)$ is BP-algebra.

2.3.1. Theorem

Suppose $(X,*,0)$ is BP-algebra, then:

i. $(0 * x) * x) = x$,
ii. $0 * (y * x) = x * y$,
iii. $x * 0 = x$,
iv. If $x * y = 0$ then $y * x = 0$,
v. If $0 * x = 0 * y$ then $x = y$,
vi. If $0 * x = y$ then $0 * y = x$,
vii. If $0 * x = x$ then $x * y = y * x$,

for all $x, y \in X$. The proof of this Theorem has been given in [25].

2.4. Definition 0-Commutative BP-Algebra

BP-algebra $(X,*,0)$ is said to be 0-commutative if $x * (0 * y) = y * (0 * x)$ for all $x, y \in X$.

2.4.1. Proposition

If $(X,*,0)$ is 0-commutative BP-algebra, then:

i. $(x * z) * (y * z) = (x * y) * (z * x)$,
ii. $x * y = (0 * y) * (0 * x)$,

for all $x, y, z \in X$. The proof of this Proposition has been given in [25].

Example, given $G = \{0,1,2,3\}$ is a set with the Cayley table as follows:

Table 2. Cayley's table for $(X,*,0)$.

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Based on Table 2, it can be proved that $(G,*,0)$ satisfies all the axioms in BP-algebra. So that, $(G,*,0)$ is BP-algebra and it can be verified that $(G,*,0)$ is 0-commutative BP-algebra.

2.4.2. Theorem

If $(X,*,0)$ is a 0-commutative B-algebra, then $(X,*,0)$ is a BP-algebra. Proof of this Theorem has been given in [2]. Let $(X,*,0)$ be a B-algebra, defined $x \wedge y = y * (y * x)$ for all $x, y \in X$.

2.5. Definition Derivation in B-Algebra

Let $(X,*,0)$ is a B-algebra and d is a self-map of X. d is said to be (l,r)-derivation in X, if it satisfies $d(x * y) = (d(x) * y) \wedge (x * d(y))$ for all $x, y \in X$, and d is said to be $(r\ ,l)$-derivation in X if it satisfies $d(x * y) = ((x * d(y) \wedge d(x) * y))$for all $x, y \in X$, and d is said to be derivation in X if it satisfies (l,r) and (r,l)-derivation in X.

2.6. Definition Regular in B-algebra

Let $(X,*,0)$ is a B-algebra and d is a self-map of X. d is called regular if it satisfies $d(0) = 0$.

3. RESULTS AND DISCUSSION

3.1. Definition 1

Let $(X,*,0)$ is a BP-algebra, for any $t \in X$ we define a d_t mapping of X itself with $d_t(x) = x * t$for each $x \in X$.

3.2. Definition 2

Let $(X,*,0)$ be a BP-algebra and d_t is a self map of X. d_t is said to be $(l,r)t$-derivation in X if it satisfies $d_t\ (x * y) = (d_t\ (x) * y\ ) \wedge (x * d_t(y))$ for all $x, y \in X$ and d_t is called $(r,l)t$-derivation in X if it satisfies $d_t(x * y) = ((x * d_t(y) \wedge (d_t(x) * y))$ for all $x, y \in X$ and d_t is said to be *t*-derivation if it satisfies $(l,r)t$-derivation and $(r,l)t$-derivation.

Example, let $G :=\ \{0,1,2,3\}$ is a BP-algebra with the following Cayley table in Table 3.

Table 3. Cayley's table for $(G,*,0)$.

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

It can be verified that $(G,*,0)$ is a BP-algebra. Then, based on the definition of 3.1 obtained

$$t = 0, d_t(x) = \begin{cases} 1, & x = 1 \\ 0, & x = 0 \\ 3, & x = 3 \\ 2, & x = 2 \end{cases} \qquad t = 1, d_t(x) = \begin{cases} 1, & x = 0 \\ 0, & x = 1 \\ 3, & x = 2 \\ 2, & x = 3 \end{cases}$$

$$t = 2, d_t(x) = \begin{cases} 1, & x = 3 \\ 0, & x = 2 \\ 3, & x = 1 \\ 2, & x = 2 \end{cases} \qquad t = 3, d_t(x) = \begin{cases} 1, & x = 2 \\ 0, & x = 3 \\ 3, & x = 0 \\ 2, & x = 1 \end{cases}$$

It can be shown that d_t is $(l,r)t$-derivation and $(r,l)t$-derivation in G, so that d_t is t-derivation in G.

3.2.1. Theorem 1

Let $(X,*,0)$ is a BP-algebra and d_t is the mapping of X to itself. If d_t is a (l,r)-t-derivation in X, then $d_t(x*y) = d_t(x)*y$ for all $x,y \in X$.

Proof,

Let $(X,*,0)$ is a BP-algebra. Since d_t is a $(l,r)t$-derivation in X and by axiom BP2 we get:

$$d_t(x*y) = (d_t(x)*y)\wedge(x*d_t(y))$$
$$d_t(x*y) = (x*d_t\,(y))*(x*d_t\,(y))*(d_t\,(x)*y))$$
$$d_t(x*y) = (d_t(x)*y)$$

hence, it is proved that $d_t(x*y) = d_t(x)*y$ for all $x,y \in X$.

The converse of Theorem 1 does hold, that is, if $d_t(x*y) = d_t(x)*y$ for all $x,y \in X$, by BP2 axiom we have:

$$d_t(x*y) = d_t(x)*y$$
$$d_t(x*y) = (x*d_t(y))*(x*d_t(y))*(d_t(x)*y))$$
$$d_t(x*y) = (d_t(x)*y))\wedge(x*d_t(y))$$

thus, it is proved that d_t is a $(l,r)t$-derivation of X.

3.2.2. Theorem 2

Let $(X,*,0)$ is a BP-algebra and d_t is a mapping of X to itself. If d_t is a $(r,l)t$-derivation in X, then $d_t(x*y) = x*d_t(y)$ for all $x,y \in X$.

Proof,

Let $(X,*,0)$ is a BP-algebra. Since d_t is a $(r,l)t$-derivation in X, then by axiom BP2 we get:

$$d_t(x*y) = (x*d_t(y))\wedge(d_t(x)*y)$$
$$d_t(x*y) = (d_t(x)*y)*((d_t(x)*y)*(x*d_t(y))$$
$$d_t(x*y) = (x*d_t(y))$$

hence, it is proved that $d_t(x*y) = x*d_t(y)$ for all $x,y \in X$.

The converse of Theorem 2 does hold, that is, if $d_t\,(x*y) = x*d_t(y)$ for all $x,y \in X$, from the BP2 axiom is obtained:

$$d_t(x*y) = x*d_t(y)$$
$$d_t(x*y) = d_t(x)*y)*((d_t(x)*y)*(x*d_t(y))$$
$$d_t(x*y) = (x*d_t(y))\wedge(d_t(x)*y)$$

thus, it is proved that d_t is a $(r,l)t$-derivation in X.

3.2.3. Proposition 1

If $(X,*,0)$ is an associative BP-algebra, then d_t is a $(r,l)t$-derivation in X.

Proof,

Let $(X,*,0)$ be an associative BP-algebra, then for all $x,y \in X$ we have:

$d_t(x*y) = (x*y)*t$
$d_t(x*y) = x*(y*t)$
$d_t(x*y) = x*d_t(y)$

from Theorem 2 it is proven that d_t is a (r,l)-t-derivation in X.

3.2.4. Proposition 2

Let $(X,*,0)$ be a BP-algebra satisfying $(x*y)*z = (x*z)*y$ for all $x,y,z \in X$, then d_t is a $(l,r)t$-derivation in X.

Proof,

Let $(X,*,0)$ is a BP-algebra that satisfies $(x*y)*z = (x*z)*y$ for all $,y,z \in X$, then:

$d_t(x*y) = (x*y)*t$
$d_t(x*y) = (x*t)*y$
$d_t(x*y) = d_t(x)*y$

from Theorem 1 it is proven that d_t is a $(l,r)t$-derivation in X.

3.3. Definition 3

A Mapping d_t of BP-algebra X is said to be t-regular if $d_t(0) = 0$.

3.3.1. Theorem

Let $(X,*,0)$ is a BP-algebra and d_t is a mapping of X to itself.

1. If d_t is a (l,r)-t-derivation in X and d_t is a t-regular, then$d_t(x) = d_t(x) \wedge x$ for all $x \in X$,

2. If d_t is a (r,l)-t-derivation in X and d_t is t-regular, then $d_t(x) = x \wedge d_t(x)$ for all $x \in X$

Proof,

Let X be a BP-algebra.

1. Let d_t be a (l,r)-t-derivation in X, since d_t is a t-regular and by Theorem (iii) in BP-Algebra subsection we get:

$d_t(x) = d_t(x*0)$
$d_t(x) = (d_t(x)*0) \wedge \big(x*d_t(0)\big)$
$d_t(x) = d_t(x) \wedge \big(x*d_t(0)\big)$
$d_t(x) = d_t(x) \wedge x*d_t(0)$
$d_t(x) = d_t(x) \wedge (x*0)$
$d_t(x) = d_t(x) \wedge x$

for all $x \in X$.

2. Let d_t be a (r,l)-t-derivation in X, since d_t is a t-regular and by Theorem (iii) in BP-Algebra subsection we get:

$$d_t(x) = d_t(x * 0)$$
$$d_t(x) = (x * d_t(x)) \wedge (d_t(x) * 0)$$
$$d_t(x) = (x * 0) \wedge (d_t(x))$$
$$d_t(x) = x \wedge d_t(x)$$

for all $x \in X$.

4. CONCLUSION

In this paper, we define a $(l,r)t$-derivation in BP-algebra and investigate its properties. Then, given the definition of $(r,l)t$-derivation in BP-algebra and investigate its properties. In general, the properties (l,r) and $(r,l)t$-derivation in BP-algebra obtained for d_t which satisfies the t-regular properties.

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