

Acoustic wave propagation model in the surface layer area based on the Runge-Kutta method

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ABSTRACT

Simulation of acoustic wave propagation in the surface layer area is carried out by taking a positive value acoustic velocity gradient so that in this simulation there will be a wave phenomenon trapped in the surface layer area. In this study, acoustic beam emission was conducted by varying the angle of incidence and length of the beam used, and the determination of the acoustic velocity gradient was carried out using the Runge-Kutta order of order 2. The results showed that at depths of 0 to 40 meters with an acoustic velocity of 1405 m/s, for a length of 1 meter and a gradient pattern of +2.5 obtained a minimum angle of incidence of 0.09 radians with a wave propagation of 250 meters. Comparison between analytical and computational results on wave propagation in the surface layer area is a 4.51% error.

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1. INTRODUCTION

Waves are vibrational energy propagations that propagate through a medium or without a medium [1, 2]. Acoustics is the science that deals with all mechanical waves in gases, liquids, and solids. Underwater acoustics is a marine field that detects targets in water areas and the bottom of the water using sound as a median [3-6].

The development of underwater acoustics has really helped humans as a tool for fishermen and the navy. Underwater acoustic waves are useful for detecting the presence of life in the sea. These waves are used to listen for underwater explosions (seismic), earthquakes, volcanic eruptions, sounds produced by fish and other animals, ship activity, or as equipment to detect underwater conditions [7-9]. These waves are also used to measure the distance from the detected object and its size, such as detecting submarines, fish species, fish size, number of fish, plankton, and other marine living creatures as well as measuring the travel time of the wave [10-14].

The surface area of the sea contains aquatic living creatures that are active, so with the presence of underwater acoustic rays, their presence can be known. With the Runge-Kutta method, acoustic rays that travel across this surface area will be reflected and refracted if they pass through a different medium. The explanation above can highlight research related to the MATLAB program to simulate the propagation of acoustic rays in the surface layer area.

2. LITERATURE REVIEW

Sound waves are longitudinal waves that occur due to compression and stretching in a gas, liquid, or solid medium. Cases such as waves on a string are simply propagated disturbances, while the molecules themselves simply vibrate back and forth around the equilibrium position. Sound waves are

divided into three categories, namely infrasonic waves with a frequency < 20 Hz, audio sonic waves with a frequency of $20 - 20,000$ Hz, and ultrasonic waves with a frequency $> 20,000$ Hz [15].

Basic acoustic theory uses several assumptions to facilitate the derivation of basic acoustic equations. The assumption used is that the fluid is inviscid or there is no frictional force between the particles. The calculations are carried out on a small scale or the changes that occur are very small when compared with the ambient value and the fluid velocity is assumed to be zero ($U_0 = 0$). The easiest way to understand the physics and dynamics of acoustic wave sound devices is through impulse response models [16].

The governing equations used in deriving the basic acoustic equations are the law of conservation of mass, the law of conservation of momentum, and the equation of state. The increase in the speed of sound under the sea is directly proportional to the increase in temperature, salinity, and depth. Salinity has little effect on the speed of sound in deep water [17]. The sound speed profile is shown in Figure 1.

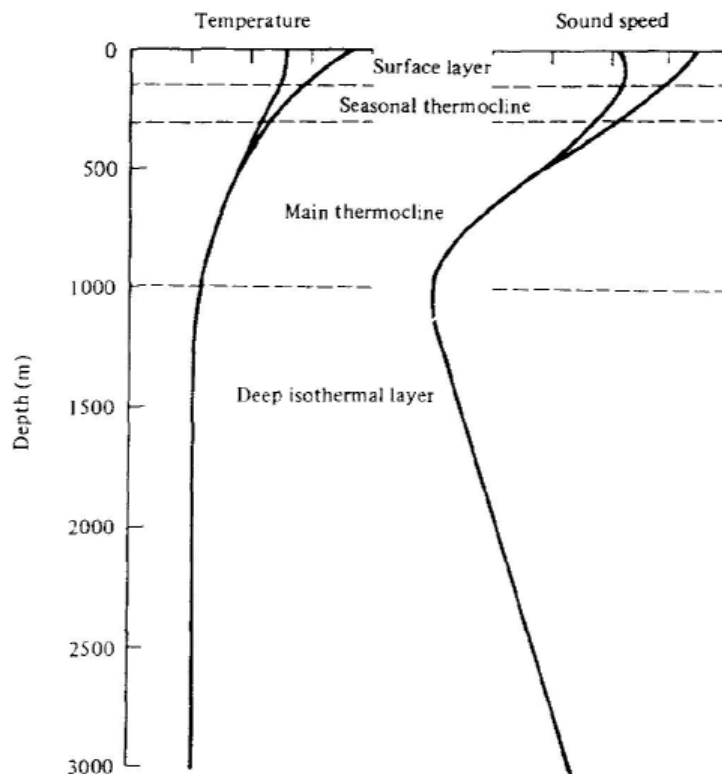


Figure 1. Sound speed profile [18].

Acoustic propagation is the transmission of acoustic energy through the intermediary medium of water. Sound waves propagate through refraction, reflection, and transmission. Trigonometry is used to see that the ratio of the cosine of an angle to the speed of sound remains constant across a boundary. This observation is called Snell's Law, resulting in the Equation:

$$\cos(\theta_1) = \frac{BD}{AD} = \frac{c_1 \Delta t}{AD} \quad (1)$$

$$\cos(\theta_2) = \frac{AE}{AD} = \frac{c_2 \Delta t}{AD} \quad (2)$$

$$\frac{\cos(\theta_1)}{c_1 \Delta t} = \frac{\cos(\theta_2)}{c_2 \Delta t} = \frac{1}{AD} \quad (3)$$

$$\frac{\cos(\theta_1)}{c_1} = \frac{\cos(\theta_2)}{c_2} \quad (4)$$

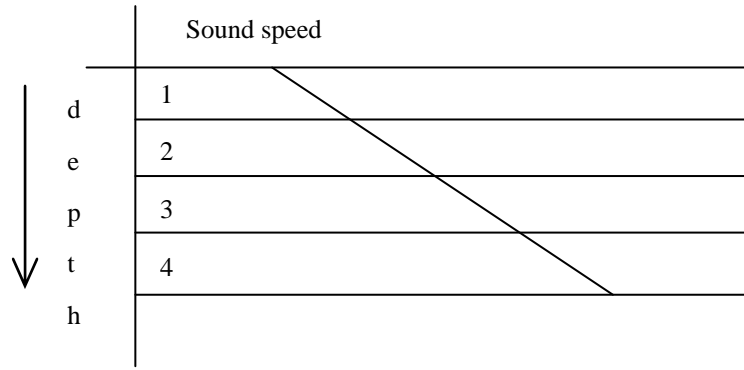


Figure 2. Speed of sound in a layered medium [19].

The speed of sound as a function of depth can be described by a simple linear equation. These results can be used to find functions for the radius of sound rays as well as other quantities.

The sound speed gradient g , shown as a dotted line, the magnitude of the sound speed in the $i + 1$ layer is shown by the following iteration equation:

$$c_{i+1} = c_i + g_i \Delta z_i \quad (5)$$

where, g is the gradient, $g = \frac{\Delta c}{\Delta z}$, from Snell's law and plugging in the equation for c , we get:

$$\frac{\cos \theta_1}{c_1} = \frac{\cos \theta}{c} \quad (6)$$

$$\frac{\cos \theta_1}{c_1} = \frac{\cos \theta}{c_1 + gz} \quad (7)$$

$$gz \cos \theta_1 = c_1 (\cos \theta - \cos \theta_1) \quad (8)$$

$$z = R (\cos \theta - \cos \theta_1) \quad (9)$$

where, $R = c_1 / g \cos \theta_1$, R is the radius of curvature of the sound ray [20].

For this case, if the surface layer has a positive gradient and the layer is deep enough, then the sound can be deflected back to the surface and then reflected back into the layer. After being reflected back down, it bends back towards the surface only to be reflected back up to the surface, as seen in Figure 3.

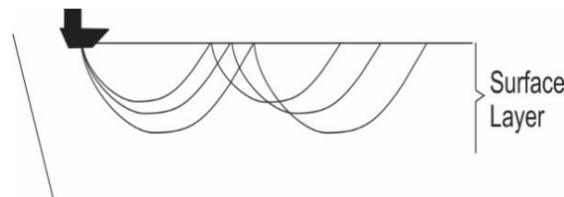


Figure 3. Propagation of positive gradient waves [17].

The surface layer contains a mixed layer that occurs during the day. In this layer, the potential temperature, humidity, and wind speed tend to remain constant with increasing height [21].

In the event that the gradient g is positive but not constant, the value of c in the $i + 1$ layer is approximated using the 2nd order Runge-Kutta method, so that Equation (5) becomes:

$$c_{i+1} = c_i + 1/2(k_1 + k_2) \quad (10)$$

where, $k_1 = g_i \Delta z_i$ and $k_2 = g_{i+1} \Delta z_i$.

3. RESEARCH METHODS

This research is modeling and computing research. This research only simulates the propagation of acoustic waves in the surface layer area. The tools used in this research were laptops and MATLAB software. The shape in this propagation model is 2-dimensional regarding the x and y axes. The research was carried out in several stages, namely determining the object from the acoustic source to be modeled, creating an iterative equation for Snell's law and the Runge-Kutta method, determining secondary data to be processed, creating a Runge-Kutta method program to determine the propagation, testing the program and carrying out analytical comparisons with computing.

4. RESULTS AND DISCUSSIONS

Results of research to simulate the propagation of sound waves in the surface layer area. This wave propagation simulation uses the Runge-Kutta method with the iterative equation of Snell's Law in a 2-dimensional computational manner on the positive x and y axes. The Runge-Kutta method functions for the iterative derivation of Snellius' law. This simulation of wave propagation uses gradient parameters, beam radius, sound speed, and depth. Analytical and computational solutions are compared to find error values. If the error value is acceptable, the discussion continues with data on sound speed versus depth [22-24]. The sound speed profile data versus depth is shown in Figure 4.

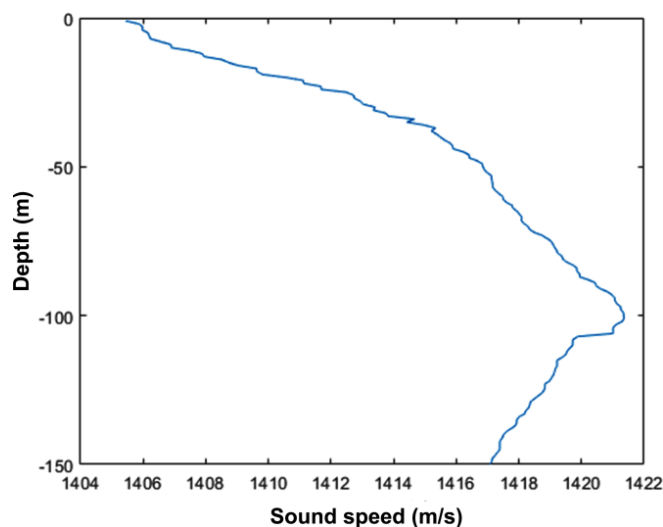


Figure 4. Sound speed profile.

This sound speed profile was created based on sound speed versus depth data obtained from the Naval Research Laboratory, Washington DC [25]. The speed of sound increases with depth, having an initial speed of 1405.44 m/s at a depth of 1 m to a depth of 101 m with a speed of 1421.36 m/s. This increase in the speed of sound occurs in a homogeneous layer which has a constant temperature and the effect of increasing pressure, so that the surface layer area has a positive gradient. The state of sound underwater has many changes, but some components are constant. Refraction occurs not only when light travels from water to air, but whenever the speed of light changes. Likewise, sound waves that cross the ocean each time experience a change in the speed of sound. Because the speed of sound changes with changes in temperature, salinity, and pressure, sound waves will refract as they move through the ocean medium.

The Runge-Kutta method and the Snellius law iteration equation are used as the initial wave propagation program. After the initial simulation is displayed, it is continued to determine the initial program conditions. The speed of sound increases with depth in the surface layer area so that this area has a positive gradient. The image is the solution of wave propagation on the x and y axes. The x-axis is for distance and the y-axis is for depth.

The solution to the circular equation for surface layer wave propagation is entered into the program to compare analytical and computational calculations. The analytical and computational comparison display can be seen in Figure 5.

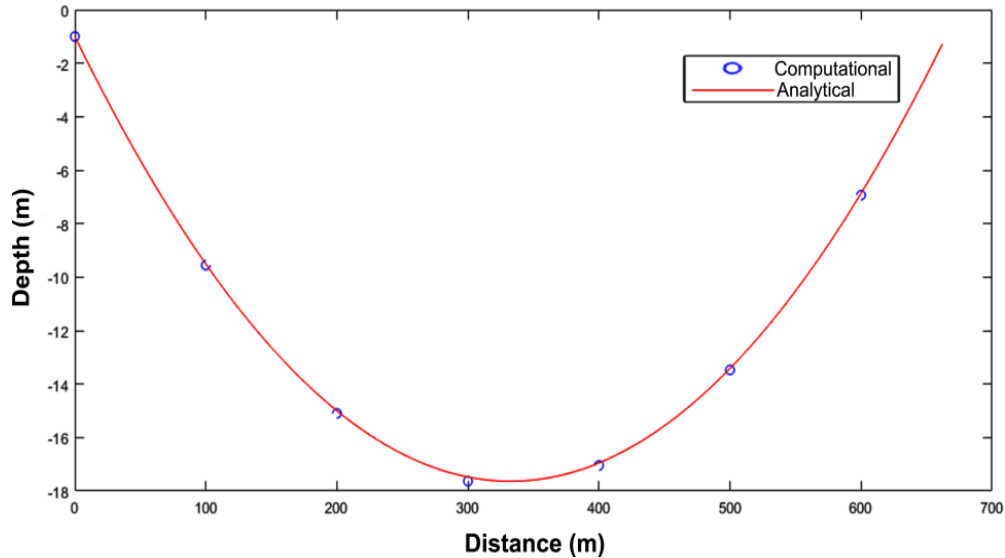


Figure 5. Comparison of analytics and computing.

Figure 5 was obtained with the initial step of determining the value of the sound speed gradient per sea depth. The center point of the wave can be determined by assuming the wave propagation is circular, so the general solution to the circular Equation is:

$$(x - x_p)^2 + (y - y_p)^2 = R^2 \quad (11)$$

from the circle Equation above we will get:

$$(y - y_p)^2 = R^2 - (x - x_p)^2 \quad (12)$$

$$y - y_p = \pm \sqrt{R^2 - (x - x_p)^2} \quad (13)$$

$$y = y_p \pm \sqrt{R^2 - (x - x_p)^2} \quad (14)$$

From Figure 5, an analytical and computational comparison can be obtained, using one analytical data, namely gradient 0.42, beam radius of 2, initial speed 1405.44 m/s, and emission angle 0.1 rad. The circle equation is used to determine the next y value from the analytical data. The iteration value obtained from analytical calculations used was 664 data. So the percentage of analytical and computational data error obtained is 0.5718%. This case of sound wave propagation with data-based gradients or changing gradients cannot be solved analytically, therefore it is solved by computational modeling.

Wave propagation in the surface layer area with variations in beam length and emission angle until transmission loss occurs using sound speed data is shown in Figure 6. The display of the wave propagation model varies using sound speed profile data. This modeling describes the propagation in the surface layer area. The difference with propagation using sound speed profile data for gradients and fixed speeds differs in the distance the beam travels at each given beam length, beam radius, and emission angle. Apart from that, wave propagation using velocity data is greatly influenced by the gradient at each depth.

Waves in the surface layer area will be reflected continuously at angles of 0.02, 0.04, 0.06, 0.08, 0.10, 0.12 rad while at angles of 0.14 rad the waves no longer reflected but will be forwarded towards the deep sound channel axis, this propagation illustrates the occurrence of Transmission Loss. The next wave size will remain the same for different beam lengths. Many other things can happen to acoustic waves as they propagate. For example, energy can scatter particles. Energy is lost when

reflected from the surface and bottom. By far the biggest factor of all is changes in propagation due to variations in speed, temperature, depth, and salinity. Changes in velocity will tend to take the form of a perfect sphere or cylinder of waves. Many conditions tend to concentrate acoustic energy resulting in loss of transmission. All these factors are called transmission loss anomalies. When the light reaches the surface, it is reflected back down and the same process starts again. Naturally, some energy is dissipated and reflected, but the overall effect is to trap the sound in a relatively small layer beneath the surface. The sound does not reach deeper areas, so transmission is reduced than expected for a cylindrical spread, this effect is called surface duct.

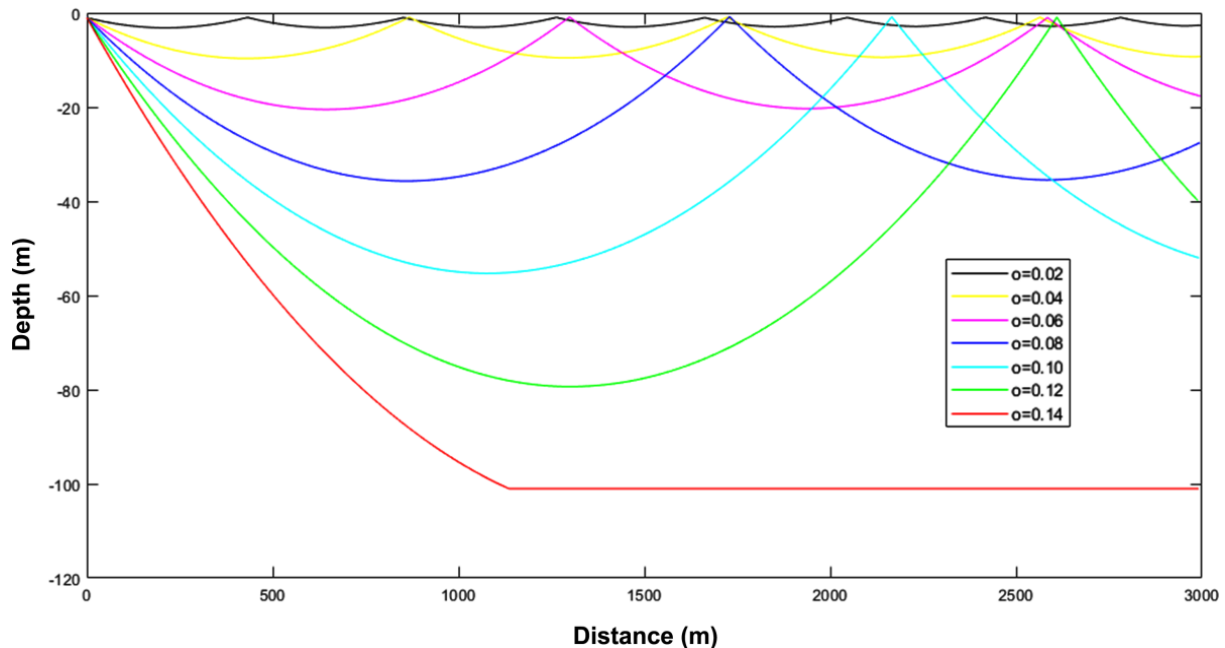


Figure 6. Wave propagation with a beam length of 1 and varying θ .

The angle of emission and radius of the beam have a big influence on wave propagation. The difference in each given beam radius is seen from the distance the beam travels. The wave propagation distance will increase with increasing beam radius. The larger the radius of the light emitted, the further the distance the light travels. Conversely, the shorter the radius of the emitted light, the shorter the distance the light will travel. All rays will be bent upwards.

5. CONCLUSION

Based on the results of research that has been carried out by modeling wave propagation in the surface layer area, it is possible to model wave propagation in the surface layer area quite well using the MATLAB application program. Modeling is carried out using the mathematical equation of Snell's law which is derived using the Runge-Kutta method. Transmission Loss from sound speed profile data below the sea surface was obtained at a depth of more than 101 m at a maximum transmit angle of 0.245 and a beam radius of 1 to 4 m. The analytical and computational difference values for the propagation of acoustic waves in the surface layer area have a beam radius of 2 m, an emission angle of 0.1, and an initial speed of 1405.44 ms^{-1} with an analytical solution using the circle equation expressed in percent, which is obtained as 0.5718%.

REFERENCES

- [1] Banerjee, A., Das, R., & Calius, E. P. (2019). Waves in structured mediums or metamaterials: A review. *Archives of Computational Methods in Engineering*, **26**, 1029–1058.
- [2] Halliday and Resnick. (2010). *Fisika dasar edisi ketujuh jilid 1 (terjemahan)*. Jakarta: Erlangga.

- [3] Surendro, B., Yuwono, N., & Darsono, S. (2015). Transmisi dan refleksi gelombang pada pemecah gelombang ambang rendah ganda tumpukan batu. *Media Komunikasi Teknik Sipil*, **20**(2), 179–187.
- [4] Diamant, R., Kipnis, D., Bigal, E., Scheinin, A., Tchernov, D., & Pinchasi, A. (2019). An active acoustic track-before-detect approach for finding underwater mobile targets. *IEEE Journal of Selected Topics in Signal Processing*, **13**(1), 104–119.
- [5] Arianto, Y., Hamdi, M., & Meyzia, B. (2022). Electrocardiogram signal patterns detection of myocardial ischemia rhythm using an artificial neural network based on MATLAB/Simulink. *Science, Technology and Communication Journal*, **3**(1), 23–32.
- [6] Defrianto, D., Putri, I. A., & Malik, U. (2022). A computational model of acoustic ray propagation in the deep-sound channel axis ocean region based on the Euler-Cromer method. *Science, Technology and Communication Journal*, **3**(1), 13–18.
- [7] Dziak, R. P., Matsumoto, H., Haver, S., Mellinger, D. K., Roche, L., Haxel, J. H., Stalin, S., Meinig, C., Kohlman, K., Sremba, A., Gedamke, J., Hatch, L., & Van Parijs, S. (2023). PMEL passive acoustics research. *Oceanography*, **36**(2/3), 196–205.
- [8] Dziak, R., Banfield, D., Lorenz, R., Matsumoto, H., Klinck, H., Dissly, R., Meinig, C., & Kahn, B. (2020). Deep ocean passive acoustic technologies for exploration of ocean and surface sea worlds in the outer solar system. *Oceanography*, **33**(2), 144–155.
- [9] Podolskiy, E. A., Murai, Y., Kanna, N., & Sugiyama, S. (2022). Glacial earthquake-generating iceberg calving in a narwhal summering ground: The loudest underwater sound in the Arctic?. *The Journal of the Acoustical Society of America*, **151**(1), 6–16.
- [10] Verfuss, U. K., Aniceto, A. S., Harris, D. V., Gillespie, D., Fielding, S., Jiménez, G., Johnston, P., Sinclair, R. R., Sivertsen, A., Solbø, S. A., Storbvold, R., Biuw, M., & Wyatt, R. (2019). A review of unmanned vehicles for the detection and monitoring of marine fauna. *Marine pollution bulletin*, **140**, 17–29.
- [11] Klemas, V. (2012). Remote sensing of environmental indicators of potential fish aggregation: An overview. *Baltica*, **25**(2), 99–112.
- [12] Korneliussen, R. J., Heggelund, Y., Macaulay, G. J., Patel, D., Johnsen, E., & Eliassen, I. K. (2016). Acoustic identification of marine species using a feature library. *Methods in Oceanography*, **17**, 187–205.
- [13] Defrianto, D., Titrawani, T., Umar, L., & Asyana, V. (2022). Identifikasi hewan berdasarkan pola akustik dengan prinsip ekstraksi wavelet dan klasifikasi multi-label jaringan syaraf tiruan. *Indonesian Physics Communication*, **19**(1), 51–56.
- [14] Fardinata, R., & Saktioto, S. (2019). Penentuan densitas spesies plasma hidrogen pada kesetimbangan termodinamik tekanan atmosfer menggunakan MATLAB. *Indonesian Physics Communication*, **16**(2), 113–117.
- [15] Tipler, P. A. (2001). *Fisika untuk Sains dan Teknik edisi ketiga jilid 1*. Jakarta: Erlangga.
- [16] Hartmann, C. S., Bell, D. T., & Rosenfeld, R. C. (1973). Impulse model design of acoustic surface-wave filters. *IEEE Transactions on Microwave Theory and Techniques*, **21**(4), 162–175.
- [17] Robert, R. J. (1983). *Principles of underwater sound 3rd edition*. Mc Grawhill: New York.
- [18] Wen, T. (2008). *Aplikasi metode normal mode pada propagasi akustik bawah air di Samudra Hindia*. Tugas Akhir, Program Studi Teknik Kelautan. Fakultas Teknik Sipil dan Lingkungan, Institut Teknologi Bandung.
- [19] Korman, M. S. (1995). *Principles of underwater sound and sonar the preliminary edition*. Dubuque, IA: Kendall/Hunt Publishing Company.
- [20] Tucholski, C. E. J. (2006). *Underwater acoustics and sonar SP411 handsout and notes fall*. United states Naval Academy: Annapolis.
- [21] Kaimal, J. C., Wyngaard, J. C., Haugen, D. A., Coté, O. R., Izumi, Y., Caughey, S. J., & Readings, C. J. (1976). Turbulence structure in the convective boundary layer. *Journal of Atmospheric Sciences*, **33**(11), 2152–2169.
- [22] Yang, W. Y. (2005). *Applied numerical methods using MATLAB*. John Wiley & Sons, Inc.
- [23] Leis, J. W. (2002). *Digital signal processing: A MATLAB - based tutorial approach*. Baldock, UK: Research Studies Press.

- [24] Oliveira, T. C., Lin, Y. T., & Porter, M. B. (2021). Underwater sound propagation modeling in a complex shallow water environment. *Frontiers in Marine Science*, **8**, 751327.
- [25] Gel Grosso, V. A. (1952). *The velocity of sound in sea water at zero depth*. Naval Research Laboratory: Washington DC.