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Decomposition and estimation of gauge transformation for Chern-Simons-Antoniadis-Savvidy forms

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ABSTRACT

Cartan's extended homotopy formula is set to obtain the Chern-Simons-Antoniadis-Savvidy (ChSAS) measurement form transformation. This indicates a change in each of the results that the quasi-invariance nature of the gauge with a closed shape under the meter transformation. The results of the initial calculation of the ChSAS form then show a singlet anomaly for each value of *n* as a dimension variation. These results have illustrated the relationship with the Chern-Simon form theory. The results of the final calculation of the ChSAS shape on the gauge transformation are then described in relation to the Zumino anomaly which reaches an anomaly variation that depends on the *p* and *n* terms.

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1. INTRODUCTION

Savvidy [1-3] has developed the extended gauge transformation to an infinite high dimension which is a non-abelian strength gauge field tensors form. It's has proved that extended gauge transformation to non-Abelian field tensor form is a large group where geometrically it can be explained as a part of the extended Lie group algebra.

Yang-Mills theory is close to the generalized for the case of independent metrics corresponding to polynomials and gauge invariance of tensor field strength to curvature forms [4-10]. The characteristics of this invariance can be used to generalize the Chern-Weil theorem to construct the invariant gauge of $(2n+2)$ form of transgression dimension [11-15]. This generalization has been used to construct high-dimensional cases known as "Chern-Simons-Antoniadis-Savvidy (ChSAS) forms" [16, 17].

This paper constructs ChSAS forms and the ChSAS transgression forms [18-20]. Furthermore, by using the Cartan homotopy formula from the ChSAS forms, it will be using gauge transform and investigates the properties of its final calculation.

2. CONTRUCTION ChSAS FORMS

Based on the gauge non-abelian tensor field form which the invariant charateristics of Pontryagin-Chern form for dimensions $(2n+3)$, it can be then simplified from $\delta\Gamma^{(2n+3)}$ form which explains the relationship of variations A and B as follows [11, 18]:

$$\delta\Gamma^{(2n+3)} = \langle F^n H \rangle, \text{ where } H = dB + [A, B] \tag{1}$$

By using Poincare Lemma and corresponding to exterior derivative $(2n+2)$ form, it gives:

$$\delta F = D(\delta A) \text{ and } \delta H = D(\delta B) + [\delta A, B] \tag{2}$$

then the variation $\delta \Gamma^{(2n+3)}$ form [11, 16, 21]:

$$\delta \Gamma^{(2n+3)} = \langle \delta F F^{n-1} H + \cdots + F^{n-1} F H + F^n \delta H \rangle = d \langle \delta A F^{n-1} H + \cdots + F^{n-1} \delta A H + F \delta B \rangle \tag{3}$$

where, t is a parameter on interval $0 \leq t \leq 1$ [5, 11]:

$$\Gamma^{(2n+3)} = \langle F^n H \rangle = dT^{(2n+2)} ChSAS \tag{4}$$

Equation (4) is known as ChSAS, then:

$$T^{(2n+2)} ChSAS(A, B) = \int_0^t dt \, \langle A F_t^{n-1} H_t + \cdots + F_t^{n-1} A H_t + F_t^n B \rangle \tag{5}$$

Equation (5) has a relation to Chern-Simos form connection 2-form dan 3-form and it can be written by using as a difference [11, 16]:

$$\langle F_1^n H_1 \rangle - \langle F_0^n H_0 \rangle = dT^{(2n+2)}(A_0, B_0, A_1, B_1) = \int_0^1 dt \, (n \langle F^{n-1} \theta H_t \rangle + \langle F_t^n \Phi \rangle) \tag{6}$$

Equation (6) is known as transgresion form af ChSAS forms [11, 16].

3. RESEARCH METHODS

We use extended Cartan homotopy formula [11, 22, 23] as follows:

$$\int_{\partial T_{r+1}} \frac{l_t^p}{p!} \pi = \int_{T_{r+1}} \frac{l_t^p}{(p+1)!} d\pi + (-1)^{p+q} \int_{T_{r+1}} \frac{p_t^{p+1}}{(p+1)!} d\pi \tag{7}$$

The exterior derivative and maps differensial form on M and T_{r+1} are given by:

$$l_t : \Omega^a(M) \times \Omega^b(T_{r+1}) \to \Omega^{a-1}(M) \times \Omega^{b+1}(T_{r+1}) \tag{8}$$

where, $\pi = \langle F_t^n H_t \rangle$ is then substituted in Equation (7):

$$\int_{dT_1} \pi = d \int_{T_1} l_t \pi \tag{9}$$

For $p = 0$ we write Equation (6) to be:

$$\langle F_1^n H_1 \rangle - \langle F_1^n H_0 \rangle = d \int_0^1 dt \, (n \langle F_t^{n-1} \theta H_t \rangle + \langle F_t^n \Phi \rangle) \tag{10}$$

For $p = 1$ we write Equation (7) to be:

$$\int_{\partial T_{p+1}} l_t^p \langle F_t^n H_t \rangle = -d \int_{T_{p+1}} \frac{l_t^2}{2} \langle F_t^n H_t \rangle \tag{11}$$

By integration on simplex:

$$d \int_{T_2} \frac{l_t^2}{2} \langle F_t^{n+1} \rangle = dT^{(2n+2)}(A_2, B_2, A_1, B_1, A_0, B_0) \tag{12}$$

$$dT^{(2n+2)}(A_2, B_2, A_1, B_1, A_0, B_0) = \langle F_1^n H_1 \rangle - \langle F_1^n H_0 \rangle \tag{13}$$

and then:

$$\langle F_1^n H_1\rangle - \langle F_1^n H_0\rangle \quad = \quad \int_0^1 dt \int_0^t ds\, \{n(n-1)\langle F_t^{n-1}\theta, \theta H_t\rangle + n\langle F_t^{n-1}\theta\Phi\rangle$$
$$+n\langle F_t^{n-1}\theta\Phi\rangle + n\langle F_t^{n-1}\theta\Phi\rangle\} \tag{14}$$

Equation (10) and (13) are extended Cartan homotopy formula in trangression form of ChSAS for $(2n+2)$ dimension.

4. RESULTS AND DISCUSSION

The application Cartan homotopy formula for extended gauge transformation ChSAS forms are writtens as follows:

$$A^g = g^{-1}Ag + g^{-1}dg = g^{-1}(A+V)g \tag{15}$$

$$B^g = g^{-1}Bg + g^{-1}dg = g^{-1}(B+V)g \tag{16}$$

From Equation (15) and (16) gauge transformation ChSAS forms can be simplified:

$$\langle F_1^n H_1\rangle - \langle F_1^n H_0\rangle = d\int_0^1 dt\, (n\langle F_t^{n-1}\theta H_t\rangle + \langle F_t^n\Phi\rangle) \tag{17}$$

By subsitution of Equation (15) and (16) in Equation (17) it gives:

$$T^{(2n+2)}(A^g, F^g) \quad = \quad n\int_0^1 dt\, tr\, (A+V)\langle (tF^g + (t^2-t)(A+V)^2)^{n-1} tH^g + (t^2-t)$$
$$([A,B]+V)\rangle + (B+V)^g\langle (tF^g + (t^2-t)(A+V)^2)^n\rangle\} \tag{18}$$

for $p=0$ Equation (18) can be shown as:

$$T^{(2n+2)}(A^g, F^g) \quad = \quad \int_0^1 dt \int_0^t ds\, tr\, \{(n(n-1)\langle F_t^{n-1}\theta, \theta H_t\rangle + n\langle F_t^{n-1}\theta\Phi\rangle$$
$$+n\langle F_t^{n-1}\theta\Phi\rangle + n\langle F_t^{n-1}\theta\Phi\rangle)\} \tag{19}$$

where, $A_t = A_0 + t(A_1 - A_0) + s(A_2 - A_1)$, $B_t = B_0 + t(B_1 - B_0) + s(B_2 - B_1)$ and $t \in [0.1]$ [16, 23, 24], using same way in Equation (19) we obtain:

$$T^{(2n+2)}(A^g, F^g) \quad = \quad n(n-1)\int_0^1 dt \int_0^t ds\, tr\langle (t(F_1 - F_0) + s(F_2 - F_1) + (t^2-t)([(A_1$$
$$+A_0 + 2A_0A_1)] + V) + (s^2 - s)([(A_2 + A_1 + 2A_0A_1)] + V) + 2st$$
$$\left([(A_0A_1 - A_0A_2 - A_1{}^2)] + V) - 2s([A_0A_1] + V)\right)\, \theta\theta t(H_1 - H_0)$$
$$+s(H_2 - H_1) + (t^2-t)([(A_0B_0 - A_0B_1 - A_1B_0 + A_1B_1)] + V) +$$
$$(s^2 - s)([(A_1B_1 - A_1B_2 - A_2B_1 + A_2B_2)] + V) + st([(A_0B_1 + A_0$$
$$B_2 + A_1B_0 + A_1B_2 + A_2B_0 + A_2B_1)] + V) + 2st([A_1B_1]) + V\rangle +$$
$$3(n\langle (t(F_1 - F_0) + s(F_2 - F_1) + (t^2-t)([(A_1 + A_0 + 2A_0A_1)] +$$

$$V) + (s^2 - s)([(A_2 + A_1 + 2A_0A_1)] + V)2st([(A_0A_1 - A_0A_2 - A_1{}^2)] + V) - 2s([A_0A_1] + V))\theta\Phi\rangle) \quad (20)$$

Equation (17) and (20) can be defined as $(2n + 2)$ form of ChSAS:

$$R^{(2n+2)} = k_{01}T^{(2n+2)}(A_t{}^g, B_t{}^g, F_t{}^g H_t{}^g) = k_{01}T^{(2n+2)}(A_t, B_t, F_t H_t) \quad (21)$$

using Equation (21) we will derive gauge transformation ChSAS forms becomes:

$$T^{(2n+2)}(A^g, B^g, F^g G^g) = T^{(2n+2)}(A, B, FH) + T^{(2n+2)}(g^{-1}dg, 0) + dR^{(2n+1)} \quad (22)$$

$$T^{(2n+2)}(A, B, FH) = T^{(2n+2)}(A, B, FH) + T^{(2n+2)}(V, 0) + dR^{(2n+1)} \quad (23)$$

Equation (23) has a corresponse to satisfy the Chern-Simons $(2n + 2)$ form. Furthermore, it is simplified for $n = 2$ so that the anomalous forms appear in the dimensions of the ChSAS singular form as follows:

$$\begin{aligned} T^{(2n+2)}(A^g, F^g) &= n\int_0^1 dt\ \ tr\ A\langle(tF + (t^2 - t)A^2)^{n-1}(tH + (t^2 - t)[A, B])\rangle \\ &= tr\left[A\left\langle\left(F - \frac{1}{3}A^2\right)\left(H - \frac{1}{3}[A, B]\right)\right\rangle + B\left\langle\frac{2}{3}F^2 - \frac{1}{2}A^2 + \frac{1}{5}A^4\right\rangle\right] \end{aligned} \quad (24)$$

Equation (24) shows results which have a relationship with Chern-Simons form $tr[A\langle(F - 1/3(A^2)\,)\rangle]$ [21, 22]. Based on the results of the gauge transformation of the ChSAS form which obtain variations form like-Zumino anomaly from Chern-Simons Form [25]. Let it have $n = 2$ and $p = 0$:

$$\begin{aligned} T^{(2n+2)}(A^g, F^g) &= n\int_0^1 dt\ tr\ (A + V)\langle(tF + (t^2 - t)(A + V)^2)^{n-1}\big((B_t + V)tH \\ &\quad + (t^2 - t)[A, B] + V)\rangle + (B_t + V)\langle(tF + (t^2 - t)(A + V)^2)^n\rangle \end{aligned} \quad (25)$$

furthermore it will calculate the Zumino anomalies form based on Equations (17) and (20) as follows:

$$\begin{aligned} R_5 &= tr\Bigg[\langle A_t V\left((B_t + V)H_t - \frac{1}{3}[A_t, B_t] + V\right) + \frac{2}{3}A_t V(B_t + [\theta, B_t])\rangle \\ &\quad + (B_t + V)\left(\frac{4}{3}A_t V + \frac{4}{5}\big(3A_t{}^2V + 3A_t V^2\big)\right)\Bigg] \end{aligned} \quad (26)$$

then by using the same method and based on Equation (20), a relationship $t^2 = s$ is obtained:

$$\begin{aligned} T^{(2n+2)}(A^g, F^g) &= \int_0^1 dt\, tr\frac{d}{dt}\Big[\langle\Big(\frac{8}{5}(A_t + V)(F_1 - F_0) + (A_t + V)\,(F_2 - F_1) + \frac{48}{35}([(A_0 \\ &\quad A_1 - A_0A_2 - A_1{}^2) + V]\big) - 2([A_0A_1] + V)\Big)\theta, \theta\frac{8}{5}(B_t + V)(H_1 - \\ &\quad H_0) + (H_2 - H_1) - \frac{3}{5}([(A_0B_0 - A_0B_1 - A_1B_0 + A_1B_1)] + V) - \frac{1}{2} \end{aligned}$$

$$([(A_1B_1 - A_1B_2 - A_2B_1 + A_2B_2)] + V) + \frac{24}{35}([(A_0B_1 + A_0B_2 + A_1$$

$$B_0 + A_1B_2 + A_2B_0 + A_2B_1)] + V) + \frac{48}{35}([A_1B_1] + V)\rangle + 3\Big(n\langle\Big(\frac{4}{5}$$

$$(A_t + V)(F_1 - F_0) + (A_t + V)(F_2 - F_1) + \frac{3}{5}([(A_1 + A_0 + 2A_0A_1)]$$

$$+V) - \frac{1}{2}([(A_2 + A_1 + 2A_0A_1)] + V)\frac{48}{35}\big([\big(A_0A_1 - A_0A_2 - {A_1}^2\big)] +$$

$$V) - 2([A_0A_1] + V)\big){A_1}^2\big)\big] + V\big) - 2s([A_0A_1] + V)\big)\theta\Phi\rangle\big)\big] \qquad (27)$$

where, Equations (26) and (27) show the results of variations in anomalies which are a form of expansion of Zumino anomalies. In this case, it shows the complexity of the calculation of the transformation gauge form in ChSAS [21, 22]. Equations (26) and (27) describe a non-abelian variations anomaly in five dimensions. This anomaly shows a tensor gauge transformation on connection 2-form.

5. CONCLUSION

The application of the Cartan homotopy formula in ChSAS shows $(A_2, B_2, A_1, B_1, A_0, B_0)$ gauge connection that explains the expansion relationship of the generalization of the Chern-Weil theorem. Based on results of the gauge transformation of ChSAS in Equations (16) and (19), they shows changes in each result which explains the gauge quasi-invariance properties under the gauge transformation. In the case of the results of calculation in Equation (17) it depicts $A_0 = 0$ and $B_0 = 0$, where this result is not invariant. The results of the Equation (23) calculation of the ChSAS forms in the form of transgressions show a single anomaly relationship for each n value. These results show a connection with the Chern-Simon forms theory shwo non-Abelian anomaly. Furthermore, the results of final calculation of the ChSAS gauge transformation form describe a relationship to Zumino's anomalies which show anomalous variations that depend on values of n and p. The calculation of Leibniz's rules becomes more complicated, this is due to the high dimensions of the ChSAS theory and the emergence of anomalous variations and increasingly nonlinear results caused by gauge transformations.

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