Sintechcom: Science, Technology, and Communication Journal
Vol. 2, No. 3, June 2022, pp. 67-72

Decomposition and Estimation of Gauge Transformation for Chern-Simons-Antoniadis-Savvidy Forms

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ABSTRACT

Cartan's extended homotopy formula is set to obtain the Chern-Simons-Antoniadis-Savvidy (ChSAS) measurement form transformation. This indicates a change in each of the results that the quasi-invariance nature of the gauge with a closed shape under the meter transformation. The results of the initial calculation of the ChSAS form then show a singlet anomaly for each value of n as a dimension variation. These results have illustrated the relationship with the Chern-Simon form theory. The results of the final calculation of the ChSAS shape on the gauge transformation are then described in relation to the Zumino anomaly which reaches an anomaly variation that depends on the p and n terms.

ARTICLE INFO

Article history:

Received Oct 23, 2021
Revised Jan 14, 2022
Accepted Feb 27, 2022

Keywords:

Cartan Homotopy
Gauge Transformation
ChSAS Foms
Gravity Anomaly

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1. INTRODUCTION

Savvidy [1-3] has developed the extended gauge transformation to an infinite high dimension which is a non-abelian strength gauge field tensors form. Its has proved that extended gauge transformation to non-Abelian field tensor form is a large group where geometrically it can be explained as a part of the extended Lie group algebra.

Yang-Mills theory is close to the generalized for the case of independent metrics corresponding to polynomials and gauge invariance of tensor field strength to curvature forms [4, 5]. The characteristics of this invariance can be used to generalize the Chern-Weil theorem to construct the invariant gauge of $(2n + 2)$ form of transgression dimension [6-10]. This generalization has been used to construct high-dimensional cases known as "Chern-Simons-Antoniadis-Savvidy (ChSAS) forms" [11, 12].

This paper constructs ChSAS forms and the ChSAS transgression forms [13-15]. Furthermore, by using the Cartan homotopy formula from the ChSAS forms, it will be using gauge transform and investigates the properties of its final calculation.

2. CONTRUCTION ChSAS FORMS

Based on the gauge non-abelian tensor field form which the invariant charateristics of Pontryagin-Chern form for dimensions $(2n + 3)$, it can be then simplified from $\delta\Gamma^{(2n+3)}$ form which explains the relationship of variations A and B as follows [6, 13]:

$$\delta\Gamma^{(2n+3)} = \langle F^n H\rangle, \text{ where } H = dB + [A, B] \tag{1}$$

By using Poincare Lemma and corresponding to exterior derivative $(2n + 2)$ form, it gives:

$$\delta F = D(\delta A) \text{ and } \delta H = D(\delta B) + [\delta A, B] \tag{2}$$

then the variation $\delta\Gamma^{(2n+3)}$ form [6, 11, 16]:

$$\delta\Gamma^{(2n+3)} = \langle \delta F F^{n-1} H + \cdots + F^{n-1} F H + F^n \delta H \rangle = d\langle \delta A F^{n-1} H + \cdots + F^{n-1} \delta A H + F \delta B \rangle \tag{3}$$

where t is a parameter on interval $0 \leq t \leq 1$ [5, 6]:

$$\Gamma^{(2n+3)} = \langle F^n H \rangle = dT^{(2n+2)} ChSAS \tag{4}$$

Equation (4) is known as ChSAS, then:

$$T^{(2n+2)} ChSAS(A, B) = \int_0^t dt\, \langle A F_t^{n-1} H_t + \cdots + F_t^{n-1} A H_t + F_t^n B \rangle \tag{5}$$

Equation (5) has a relation to Chern-Simos form connection 2-form dan 3-form and it can be written by using as a difference [6, 11]:

$$\langle F_1^n H_1 \rangle - \langle F_0^n H_0 \rangle = dT^{(2n+2)}(A_0, B_0, A_1, B_1) = \int_0^1 dt\, (n\langle F^{n-1} \theta H_t \rangle + \langle F_t^n \Phi \rangle) \tag{6}$$

Equation (6) is known as transgresion form af Chern-Simons-Antoniadis-Savvidy forms [6, 11].

3. RESEARCH METHODS

We use extended Cartan homotopy formula [6, 17, 18] as follows:

$$\int_{\partial T_{r+1}} \frac{l_t^p}{p!} \pi = \int_{T_{r+1}} \frac{l_t^p}{(p+1)!} d\pi + (-1)^{p+q} \int_{T_{r+1}} \frac{p_t^{p+1}}{(p+1)!} d\pi \tag{7}$$

The exterior derivative and maps differensial form on M and T_{r+1} are given by:

$$l_t : \Omega^a(M) \times \Omega^b(T_{r+1}) \to \Omega^{a-1}(M) \times \Omega^{b+1}(T_{r+1}) \tag{8}$$

where $\pi = \langle F_t^n H_t \rangle$ is then substituted in Equation (7):

$$\int_{dT_1} \pi = d \int_{T_1} l_t \pi \tag{9}$$

For $p = 0$ we write Equation (6) to be:

$$\langle F_1^n H_1 \rangle - \langle F_1^n H_0 \rangle = d \int_0^1 dt\, (n\langle F_t^{n-1} \theta H_t \rangle + \langle F_t^n \Phi \rangle) \tag{10}$$

For $p = 1$ we write Equation (7) to be:

$$\int_{\partial T_{p+1}} l_t^p \langle F_t^n H_t \rangle = -d \int_{T_{p+1}} \frac{l_t^2}{2} \langle F_t^n H_t \rangle \tag{11}$$

By integration on simplex:

$$d \int_{T_2} \frac{l_t^2}{2} \langle F_t^{n+1} \rangle = dT^{(2n+2)}(A_2, B_2, A_1, B_1, A_0, B_0) \tag{12}$$

$$dT^{(2n+2)}(A_2, B_2, A_1, B_1, A_0, B_0) = \langle F_1^n H_1 \rangle - \langle F_1^n H_0 \rangle \tag{13}$$

and then:

$$\begin{aligned} \langle F_1^n H_1 \rangle - \langle F_1^n H_0 \rangle \quad = \quad & \int_0^1 dt \int_0^t ds \, \{ n(n-1) \langle F_t^{n-1} \theta, \theta H_t \rangle + n \langle F_t^{n-1} \theta \Phi \rangle \\ & + n \langle F_t^{n-1} \theta \Phi \rangle + n \langle F_t^{n-1} \theta \Phi \rangle \} \end{aligned} \tag{14}$$

Equation (10) and (13) are extended Cartan homotopy formula in trangression form of ChSAS for $(2n+2)$ dimension.

4. RESULTS AND DISCUSSION

The application Cartan homotopy formula for extended gauge transformation ChSAS forms are writtens as follows:

$$A^g = g^{-1} A g + g^{-1} dg = g^{-1}(A + V) g \tag{15}$$

$$B^g = g^{-1} B g + g^{-1} dg = g^{-1}(B + V) g \tag{16}$$

From Equation (15) and (16) gauge transformation ChSAS forms can be simplified:

$$\langle F_1^n H_1 \rangle - \langle F_1^n H_0 \rangle = d \int_0^1 dt \, (n \langle F_t^{n-1} \theta H_t \rangle + \langle F_t^n \Phi \rangle) \tag{17}$$

By subsitution of Equation (15) and (16) in Equation (17) it gives:

$$\begin{aligned} T^{(2n+2)}(A^g, F^g) \quad = \quad & n \int_0^1 dt \, tr \, (A + V) \langle (tF^g + (t^2 - t)(A + V)^2)^{n-1} t H^g + (t^2 - t) \\ & ([A, B] + V) \rangle + (B + V)^g \langle (tF^g + (t^2 - t)(A + V)^2)^n \rangle \} \end{aligned} \tag{18}$$

for $p = 0$ Equation (18) can be shown as:

$$T^{(2n+2)}(A^g,F^g) = \int_0^1 dt\int_0^t ds\ tr\ \{(n(n-1)\langle F_t^{n-1}\theta,\theta H_t\rangle + n\langle F_t^{n-1}\theta\Phi\rangle$$
$$+n\langle F_t^{n-1}\theta\Phi\rangle + n\langle F_t^{n-1}\theta\Phi\rangle)\} \quad (19)$$

where $A_t = A_0 + t(A_1 - A_0) + s(A_2 - A_1)$, $B_t = B_0 + t(B_1 - B_0) + s(B_2 - B_1)$ and $t \in [0.1]$ [11, 18, 19], using same way in Equation (19) we obtain:

$$T^{(2n+2)}(A^g,F^g) = n(n-1)\int_0^1 dt\int_0^t ds\ tr\langle(t(F_1-F_0)+s(F_2-F_1)+(t^2-t)([(A_1$$
$$+A_0+2A_0A_1)]+V)+(s^2-s)([(A_2+A_1+2A_0A_1)]+V)+2st$$
$$\left([\left(A_0A_1-A_0A_2-{A_1}^2\right)]+V\right)-2s([A_0A_1]+V)\Big)\ \theta\theta t(H_1-H_0)$$
$$+s(H_2-H_1)+(t^2-t)([(A_0B_0-A_0B_1-A_1B_0+A_1B_1)]+V)+$$
$$(s^2-s)([(A_1B_1-A_1B_2-A_2B_1+A_2B_2)]+V)+st([(A_0B_1+A_0$$
$$B_2+A_1B_0+A_1B_2+A_2B_0+A_2B_1)]+V)+2st([A_1B_1])+V\rangle+$$
$$3(n\langle(t(F_1-F_0)+s(F_2-F_1)+(t^2-t)([(A_1+A_0+2A_0A_1)]+$$
$$V)+(s^2-s)([(A_2+A_1+2A_0A_1)]+V)2st([(A_0A_1-A_0A_2-$$
$${A_1}^2)]+V\big)-2s([A_0A_1]+V)\big)\theta\Phi\rangle\big) \quad (20)$$

Equation (17) and (20) can be defined as $(2n+2)$ form of ChSAS:

$$R^{(2n+2)} = k_{01}T^{(2n+2)}({A_t}^g,{B_t}^g,{F_t}^g{H_t}^g) = k_{01}T^{(2n+2)}(A_t,B_t,F_tH_t) \quad (21)$$

using Equation (21) we will derive gauge transformation ChSAS forms becomes:

$$T^{(2n+2)}(A^g,B^g,F^gG^g) = T^{(2n+2)}(A,B,FH) + T^{(2n+2)}(g^{-1}dg,0) + dR^{(2n+1)} \quad (22)$$

$$T^{(2n+2)}(A,B,FH) = T^{(2n+2)}(A,B,FH) + T^{(2n+2)}(V,0) + dR^{(2n+1)} \quad (23)$$

Equation (23) has a corresponse to satisfy the Chern-Simons $(2n+2)$ form. Furthermore, it is simplified for $n = 2$ so that the anomalous forms appear in the dimensions of the ChSAS singular form as follows:

$$T^{(2n+2)}(A^g,F^g) = n\int_0^1 dt\ \ tr\ A\langle(tF+(t^2-t)A^2)^{n-1}(tH+(t^2-t)[A,B])\rangle$$
$$= tr\left[A\left\langle\left(F-\frac{1}{3}A^2\right)\left(H-\frac{1}{3}[A,B]\right)\right\rangle + B\left\langle\frac{2}{3}F^2-\frac{1}{2}A^2+\frac{1}{5}A^4\right\rangle\right] \quad (24)$$

Equation (24) shows results which have a relationship with Chern-Simons form $tr\left[A\left\langle\left(F-\frac{1}{3}A^2\right)\right\rangle\right]$ [16, 17]. Based on the results of the gauge transformation of the ChSAS form which obtain variations form like-Zumino anomaly from Chern-Simons Form [20]. Let it have $n = 2$ and $p = 0$:

$$T^{(2n+2)}(A^g, F^g) = n\int_0^1 dt\, tr\,(A+V)\langle(tF+(t^2-t)(A+V)^2)^{n-1}\big((B_t+V)tH$$

$$+(t^2-t)[A,B]+V)\rangle + (B_t+V)\langle(tF+(t^2-t)(A+V)^2)^n\rangle \qquad (25)$$

furthermore it will calculate the Zumino anomalies form based on Equations (17) and (20) as follows:

$$R_5 = tr\Bigg[\langle A_t V\left((B_t+V)H_t - \frac{1}{3}[A_t, B_t] + V\right) + \frac{2}{3}A_t V(B_t + [\theta, B_t])\rangle$$

$$+(B_t+V)\left(\frac{4}{3}A_t V + \frac{4}{5}\left(3A_t{}^2 V + 3A_t V^2\right)\right)\Bigg] \qquad (26)$$

then by using the same method and based on Equation (20), a relationship $t^2 = s$ is obtained:

$$T^{(2n+2)}(A^g, F^g) = \int_0^1 dt\, tr\frac{d}{dt}\Big[\langle\Big(\frac{8}{5}(A_t+V)(F_1-F_0) + (A_t+V)\,(F_2-F_1) + \frac{48}{35}([(A_0$$

$$A_1 - A_0A_2 - A_1{}^2) + V]) - 2([A_0A_1]+V)\Big)\theta, \theta\frac{8}{5}(B_t+V)(H_1 -$$

$$H_0) + (H_2 - H_1) - \frac{3}{5}([(A_0B_0 - A_0B_1 - A_1B_0 + A_1B_1)] + V) - \frac{1}{2}$$

$$([(A_1B_1 - A_1B_2 - A_2B_1 + A_2B_2)] + V) + \frac{24}{35}([(A_0B_1 + A_0B_2 + A_1$$

$$B_0 + A_1B_2 + A_2B_0 + A_2B_1)] + V) + \frac{48}{35}([A_1B_1] + V)\rangle + 3\Big(n\langle\Big(\frac{4}{5}$$

$$(A_t+V)(F_1-F_0) + (A_t+V)(F_2-F_1) + \frac{3}{5}([(A_1 + A_0 + 2A_0A_1)]$$

$$+V) - \frac{1}{2}([(A_2 + A_1 + 2A_0A_1)] + V)\frac{48}{35}\big([(A_0A_1 - A_0A_2 - A_1{}^2)] +$$

$$V) - 2([A_0A_1] + V)\big)A_1{}^2)\big] + V\big) - 2s([A_0A_1] + V)\big)\theta\Phi\rangle)\Big] \qquad (27)$$

where Equations (26) and (27) show the results of variations in anomalies which are a form of expansion of Zumino anomalies. In this case, it shows the complexity of the calculation of the transformation gauge form in ChSAS [16, 17]. Equations (26) and (27) describe a non-abelian variations anomaly in five dimensions. This anomaly shows a tensor gauge transformation on connection 2-form.

5. CONCLUSION

The application of the Cartan homotopy formula in ChSAS shows $(A_2, B_2, A_1, B_1, A_0, B_0)$ gauge connection that explains the expansion relationship of the generalization of the Chern-Weil theorem. Based on results of the gauge transformation of ChSAS in Equations (16) and (19), they shows changes in each result which explains the gauge quasi-invariance properties under the gauge transformation. In the case of the results of calculation in Equation (17) it depicts $A_0 = 0$ and $B_0 = 0$, where this result is not invariant. The results of the Equation (23) calculation of the ChSAS forms in the form of transgressions show a single anomaly relationship for each n value. These results show a connection with the Chern-Simon forms theory shwo non-Abelian anomaly. Furthermore, the results of final calculation of the ChSAS gauge transformation form describe a relationship to Zumino's anomalies which show anomalous variations that depend on values of n and p. The calculation of Leibniz's rules becomes more complicated, this is due to the high dimensions of the ChSAS theory and the emergence of anomalous variations and increasingly nonlinear results caused by gauge transformations.

ACKNOWLEDGMENTS

We would like to thank The Ministry of Research, Technology and Higher Education through University of Riau for generous support in this research.

REFERENCES

[1] G. Savvidy, "Non-Abelian tensor gauge fields I," *International Journal of Modern Physics A*, vol. 21, pp. 4931-4957, Sep 2006.
[2] G. Savvidy, "Non-Abelian tensor gauge fields II," *International Journal of Modern Physics A*, vol. 21, pp. 4959-4977, Sep 2006 .
[3] G. Savvidy, "Non-Abelian tensor gauge fields: generalization of Yang–Mills theory," *Physics Letters B*, vol. 625, pp. 341-350, Oct 2005.
[4] G. Savvidy, "Extension of the Poincaré group and non-Abelian tensor gauge fields," *International Journal of Modern Physics A*, vol. 25, pp. 5765-5785, Dec 2010.
[5] F. Izaurieta, I. Muñoz, and P. Salgado, "A Chern–Simons gravity action in d= 4," *Physics Letters B*, vol. 750, pp. 39-44, Nov 2015.
[6] F. Izaurieta, P. Salgado, and S. Salgado, "Chern–Simons–Antoniadis–Savvidy forms and standard supergravity," *Physics Letters B*, vol. 767, pp. 360-365, Apr 2017.
[7] S. Hamdan, E. Erwin, and S. Saktioto, "Chern-Simons-Antoniadis-Savvidy Forms and Non-Abelian Anomaly," *Journal of Aceh Physics Society*, vol. 8, pp. 11-15, Jan 2019.
[8] S. Yani, "Analisa Distribusi Dosis pada Fantom Homogen dan Inhomogen dengan Simulasi Monte Carlo," *Komunikasi Fisika Indonesia*, vol. 19, pp. 39-44, Mar 2021.
[9] T. F. Siswanti and S. Gemawati, "t-Derivations in BP-Algebras," *Science, Technology & Communication Journal*, vol. 1, pp. 97-103, Jun 2021.
[10] S. Afriastuti, S. Gemawati, and S. Syamsudhuha, "On (f, g)-Derivation in BCH-Algebra," *Science, Technology & Communication Journal*, vol. 1, pp. 87-91, Jun 2021.
[11] P. Catalán, *et al.*, "Topological gravity and Chern–Simons forms in d= 4," *Physics Letters B*, vol. 751, pp. 205-208, Dec 2015.
[12] S. Hamdan, *et al.*, "Topological Gravity of Chern-Simons-Antoniadis-Savvidy in 2+ 1 Dimensions," *Journal of Aceh Physics Society*, vol. 9, pp. 65-71, Sep 2020.
[13] R. A. Bertlmann, "Anomalies in quantum field theory," *Oxford University Press*, vol. 91, Nov 2000.
[14] M. Nakara, "Geometry, Topology and Physics," *IOP Publishing Ltd*, 2003.
[15] T. Restianingsih, "Batas Medan Lemah pada Gravitasi f (T)," *Komunikasi Fisika Indonesia*, vol. 19, pp. 25-30, Mar 2021.
[16] P. Salgado and S. Salgado, "Extended gauge theory and gauged free differential algebras," *Nuclear Physics B*, vol. 926, pp. 179-199, Jan 2018.
[17] F. Izaurieta, E. Rodríguez, and P. Salgado, "On Transgression Forms and Chern--Simons (Super) gravity," *arXiv preprint hep-th/0512014*, Dec 2005.
[18] F. Izaurieta, E. Rodríguez, and P. Salgado, "The extended Cartan homotopy formula and a subspace separation method for Chern–Simons theory," *Letters in Mathematical Physics*, vol. 80, pp. 127-138, May 2007.
[19] B. Zumino, W. Yong-Shi, and A. Zee, "Chiral anomalies, higher dimensions, and differential geometry," *Nuclear Physics B*, vol. 239, pp. 477-507, Jul 1984.
[20] A. Alekseev, *et al.*, Chern–Simons, "Wess–Zumino and other cocycles from Kashiwara–Vergne and associators," *Letters in Mathematical Physics*, vol. 108, pp. 757-778, Mar 2018.